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Homogenization of linear bianisotropic particulate composite media – Numerical studies

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Abstract. We present here the application of the Maxwell Garnett (MG) and the Bruggeman (Br) formalisms to homogenize very general linear bianisotropic-in-bianisotropic particulate composite media with ellipsoidal inclusions. Both formalisms involve the calculation of certain depolarization dyadics, which generally amounts to the numerical evaluation of several two-dimensional integrals. The MG estimate of the constitutive dyadic of the homogenized composite medium can then be obtained by straightforward matrix manipulations. The Br estimate, however, involves nonlinear equations which have to be solved iteratively. We present an iteration scheme that converged rapidly in most cases tested. Numerical results are given for those composite media for which the Bruggeman formalism so far has not been implemented: spherical chiral inclusions in a uniaxial dielectric host medium, spheroidal chiral inclusions in free space, spherical chiral inclusions in a biaxial dielectric host medium, and spherical voids in a gyrotropic dielectric host medium.

1. Introduction

The Maxwell Garnett and the Bruggeman formalisms for homogenizing isotropic dielectric–magnetic particulate composite media have been known for a long time [1], and were further developed for particulate composite media with chiral host media during the last decade [2–6]. But the extension of either homogenization formalism to particulate composite media with general bianisotropic host media was hindered by the fact that the analytical structure of the so-called depolarization dyadics for bianisotropic media was not known until recently, even for simple choices of the exclusion region geometry.

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The situation changed rapidly during the last three years. Berthier [7] theoretically homogenized ellipsoidal dielectric inclusions in an anisotropic dielectric medium that can be affinely transformed into an isotropic medium. More significantly, several different approaches for the depolarization dyadics of convex-shaped exclusion regions in anisotropic dielectric and related media became available [8–14]. Finally, Michel and Weiglhofer [15] obtained the depolarization dyadic of an ellipsoidal exclusion region in a bianisotropic host medium. The depolarization dyadic emerged as a two-dimensional integral. It can be determined numerically, but analytical simplification is also possible in special cases. Based on this result, the Maxwell Garnett (MG) and the Bruggeman (Br) formalisms for a wide class of particulate composite media with bianisotropic host media were set up by us [16].

Our aim in this paper is to present numerical studies demonstrating the versatility of the two formalisms. A fundamental, if not *the* fundamental, problem in this enterprise is the considerable complexity of bianisotropic composite media, illustrated most impressively by the large parameter space.

A general bianisotropic medium is described by four 3×3 constitutive dyadics (or, alternatively, a single 6×6 constitutive dyadic) involving 36 complex-valued parameters. An algebraic constraint [17] reduces that number by unity, thus leaving 70 real-valued constitutive parameters. In addition, there are five geometrical parameters, the ellipsoidal inclusions being characterized by two axial ratios for the shape and three Eulerian angles for the orientation. We note that the absolute size of the inclusions is irrelevant, but that the inclusions must be electrically small [18]. Finally, adding the inclusion medium concentration, we find that a composite medium comprising inclusions of one bianisotropic medium in a bianisotropic host medium therefore requires a total of $2 \times 70 + 5 + 1 = 146$ real-valued parameters for its description in the MG formalism. The Br formalism may require 5 additional parameters, because the exclusion regions for the host and the inclusion media can be ellipsoids of different shapes and orientations; see [20].

This abundance of parameters can, however, be drastically reduced if certain symmetries are present in the composite medium, e.g., if the medium is uniaxial. Only then does it appear that we have a reasonable chance for extracting simple and physically significant statements from numerical calculations. Therefore, we are confined here to bianisotropic particulate composite media that are simple enough to be described by a few parameters on the one hand, but non-trivial on the other hand in the sense that the Bruggeman formalism has not been implemented on them.

The paper is organized as follows: in Section 2, the basic equations of the MG and the Br formalisms are stated for a composite medium comprising identical, similarly oriented, randomly dispersed, bianisotropic ellipsoidal inclusions in a bianisotropic host medium. The surface of the chosen inclusion can be parametrized as

$$\underline{x}_s(\theta,\phi) = \delta\, \underline{\underline{U}} \cdot \hat{\underline{x}}(\theta,\phi), \tag{1}$$

where $\hat{\underline{x}}(\theta,\phi)$ is the radial unit vector in a spherical coordinate system located at the centroid of the inclusion, δ is a linear measure of the inclusion size, and the shape dyadic $\underline{\underline{U}}$ is real symmetric with positive eigenvalues $0 < a_j \leqslant 1$ $(j = 1,2,3)$. Numerical implementation of both formalisms is discussed in Section 3. In Section 4, numerical results are presented for selected bianisotropic-in-bianisotropic composite media.

2. Formalisms

Let the electromagnetic response properties of a bianisotropic medium be described in six-vector notation by the frequency-dependent 6×6 *constitutive dyadic* [15]

$$\underline{\underline{C}}^{\alpha} = \begin{pmatrix} \underline{\underline{\epsilon}}^{\alpha} & \underline{\underline{\xi}}^{\alpha} \\ \underline{\underline{\zeta}}^{\alpha} & \underline{\underline{\mu}}^{\alpha} \end{pmatrix}, \tag{2}$$

where $\underline{\underline{\epsilon}}^{\alpha}$ and $\underline{\underline{\mu}}^{\alpha}$ are the permittivity and permeability dyadics, whereas $\underline{\underline{\xi}}^{\alpha}$ and $\underline{\underline{\zeta}}^{\alpha}$ are the two magnetoelectric dyadics; the identifier α may take the values in/out for the inclusion/host medium or MG/Br for the Maxwell Garnett/Bruggeman estimates of the homogenized composite medium (HCM).

As the detailed derivations of both the MG and Br formalisms are available elsewhere [16], we provide here only a brief description of the terms involved. Let us begin with the 6×6 polarizability dyadics

$$\underline{\underline{a}}^{\alpha'/\alpha} = \left(\underline{\underline{C}}^{\alpha'} - \underline{\underline{C}}^{\alpha}\right) \cdot \left[\underline{\underline{I}} + \mathrm{i}\omega \underline{\underline{D}}^{\alpha'/\alpha} \cdot \left(\underline{\underline{C}}^{\alpha'} - \underline{\underline{C}}^{\alpha}\right)\right]^{-1}, \tag{3}$$

where $\underline{\underline{I}}$ is the 6×6 identity dyadic and α'/α may be each of the combinations in/out, in/Br, and out/Br; ω is the angular frequency, and an $\exp(-\mathrm{i}\omega t)$ time-dependence is implicit.

The 6×6 depolarization dyadic $\underline{\underline{D}}^{\alpha'/\alpha}$ of an ellipsoidal exclusion region in medium α is written as

$$\underline{\underline{D}}^{\alpha'/\alpha} = \begin{pmatrix} \underline{\underline{D}}_{\mathrm{ee}}^{\alpha'/\alpha} & \underline{\underline{D}}_{\mathrm{em}}^{\alpha'/\alpha} \\ \underline{\underline{D}}_{\mathrm{me}}^{\alpha'/\alpha} & \underline{\underline{D}}_{\mathrm{mm}}^{\alpha'/\alpha} \end{pmatrix}, \tag{4}$$

whose 3×3 dyadic components are given by

$$\underline{\underline{D}}_{\lambda\lambda'}^{\alpha'/\alpha} = \underline{\underline{U}}^{-1} \cdot \widetilde{\underline{\underline{D}}}_{\lambda\lambda'}^{\alpha'/\alpha} \cdot \left(\underline{\underline{U}}^{-1}\right)^{\mathrm{T}} \quad (\lambda, \lambda' = \mathrm{e}, \mathrm{m}), \tag{5}$$

with $(\underline{\underline{U}}^{-1})^{\mathrm{T}}$ denoting the transpose of $\underline{\underline{U}}^{-1}$. The 3×3 dyadic $\widetilde{\underline{\underline{D}}}_{\lambda\lambda'}^{\alpha'/\alpha}$ is calculated as the integral

$$\widetilde{\underline{\underline{D}}}_{\lambda\lambda'}^{\alpha'/\alpha} = \frac{1}{4\pi \mathrm{i}\omega} \int_{\phi=0}^{2\pi} \mathrm{d}\phi \int_{\theta=0}^{\pi} \mathrm{d}\theta \, \sin\theta \times \frac{(\hat{\underline{x}} \cdot \underline{\underline{\tau}}_{\lambda\lambda'} \cdot \hat{\underline{x}})\, \hat{\underline{x}}\hat{\underline{x}}}{(\hat{\underline{x}} \cdot \widetilde{\underline{\underline{\epsilon}}}^{\alpha} \cdot \hat{\underline{x}})(\hat{\underline{x}} \cdot \widetilde{\underline{\underline{\mu}}}^{\alpha} \cdot \hat{\underline{x}}) - (\hat{\underline{x}} \cdot \widetilde{\underline{\underline{\zeta}}}^{\alpha} \cdot \hat{\underline{x}})(\hat{\underline{x}} \cdot \widetilde{\underline{\underline{\xi}}}^{\alpha} \cdot \hat{\underline{x}})} \quad (\lambda, \lambda' = \mathrm{e}, \mathrm{m}), \tag{6}$$

where

$$\widetilde{\underline{\underline{\beta}}} = \left(\underline{\underline{U}}^{-1}\right)^{\mathrm{T}} \cdot \underline{\underline{\beta}} \cdot \underline{\underline{U}}^{-1} \quad (\underline{\underline{\beta}} = \underline{\underline{\epsilon}}^{\alpha}, \underline{\underline{\xi}}^{\alpha}, \underline{\underline{\zeta}}^{\alpha}, \underline{\underline{\mu}}^{\alpha}), \tag{7}$$

and the abbreviations

$$\underline{\underline{\tau}}_{\mathrm{ee}} = \widetilde{\underline{\underline{\mu}}}^{\alpha}, \quad \underline{\underline{\tau}}_{\mathrm{em}} = -\widetilde{\underline{\underline{\xi}}}^{\alpha}, \quad \underline{\underline{\tau}}_{\mathrm{me}} = -\widetilde{\underline{\underline{\zeta}}}^{\alpha}, \quad \underline{\underline{\tau}}_{\mathrm{mm}} = \widetilde{\underline{\underline{\epsilon}}}^{\alpha}. \tag{8}$$

The *Maxwell Garnett estimate* $\underline{\underline{C}}^{\mathrm{MG}}$ of the constitutive dyadic of the HCM is then given by

$$\underline{\underline{C}}^{\mathrm{MG}} = \underline{\underline{C}}^{\mathrm{out}} + f\underline{\underline{a}}^{\mathrm{in/out}} \cdot \left(\underline{\underline{I}} - \mathrm{i}\omega f \underline{\underline{D}}^{\mathrm{in/out}}_{\mathrm{sphere}} \cdot \underline{\underline{a}}^{\mathrm{in/out}}\right)^{-1}, \tag{9}$$

where f, $0 \leqslant f \leqslant 1$, is the inclusion volume concentration; $\underline{\underline{D}}^{\mathrm{in/out}}_{\mathrm{sphere}}$ on the right side of Eq. (9) is the depolarization dyadic of the Lorentzian cavity in the host medium. The shape of this cavity is chosen spherical, which is fully consistent with the homogeneous and isotropic dispersal of inclusions in the composite medium [16]. The depolarization dyadic $\underline{\underline{D}}^{\mathrm{in/out}}_{\mathrm{sphere}}$ is obtained from Eqs (4)–(8) by setting $\underline{\underline{U}} = \underline{\underline{I}}$, the 3×3 identity dyadic.

The *Bruggeman estimate* $\underline{\underline{C}}^{\mathrm{Br}}$ has to be obtained by solving the equation

$$f\underline{\underline{a}}^{\mathrm{in/Br}} + (1-f)\underline{\underline{a}}^{\mathrm{out/Br}} = \underline{\underline{0}}, \tag{10}$$

which contains $\underline{\underline{C}}^{\mathrm{Br}}$ implicitly through the 6×6 polarizability dyadics $\underline{\underline{a}}^{\mathrm{in/Br}}$ and $\underline{\underline{a}}^{\mathrm{out/Br}}$. Hidden in the foregoing equations is the assumption that the host medium has an ellipsoidal topology which is the same as the inclusion medium's; i.e., the same shape dyadic $\underline{\underline{U}}$ is used to compute $\underline{\underline{D}}^{\mathrm{out/Br}}$ and $\underline{\underline{D}}^{\mathrm{in/Br}}$. As we have shown elsewhere [20], the two topologies need not be the same.

On decomposing $\underline{\underline{\tau}}_{\lambda\lambda'}$ into symmetric and skew-symmetric parts,

$$\underline{\underline{\tau}}_{\lambda\lambda'} = \underline{\underline{\tau}}^{\mathrm{sym}}_{\lambda\lambda'} + \underline{\underline{\tau}}^{\mathrm{skew}}_{\lambda\lambda'}, \tag{11}$$

we can clearly see that $\underline{\underline{D}}^{\mathrm{in/out}}$ and $\underline{\underline{D}}^{\mathrm{in/out}}_{\mathrm{sphere}}$ depend on $\underline{\underline{\tau}}^{\mathrm{sym}}_{\lambda\lambda'}$ but not on $\underline{\underline{\tau}}^{\mathrm{skew}}_{\lambda\lambda'}$. In general, the integral on the right side of Eq. (6) must be calculated numerically, at least in the current stage of the theoretical development. But $\underline{\underline{D}}^{\mathrm{in/out}}$ and $\underline{\underline{D}}^{\mathrm{in/out}}_{\mathrm{sphere}}$ can be obtained analytically if the symmetric parts $\underline{\underline{\tau}}^{\mathrm{sym}}_{\lambda\lambda'}$ are uniaxial – i.e., when

$$\underline{\underline{\tau}}^{\mathrm{sym}}_{\lambda\lambda'} = \tau^{\mathrm{sym}}_{\lambda\lambda'\mathrm{t}}\left[\underline{\underline{I}} - \underline{c}\,\underline{c}\right] + \tau^{\mathrm{sym}}_{\lambda\lambda'\mathrm{c}}\,\underline{c}\,\underline{c}, \tag{12}$$

where $\underline{c}$ is a unit vector along the crystallographic axis of the medium. This specialization guarantees a drastic reduction of the parameter space, even though it still spans a rich variety of complex linear media: chiral media, uniaxial dielectric/magnetic media, uniaxial bianisotropic media, gyrotropic and gyrotropic-like bianisotropic media.

3. Numerical implementation

The MG equation (9) only contains the depolarization dyadics of spherical and ellipsoidal exclusion regions in the host medium. Therefore, $\underline{\underline{C}}^{\mathrm{MG}}$ can be obtained analytically, if $\underline{\underline{D}}^{\mathrm{in/out}}$ and $\underline{\underline{D}}^{\mathrm{in/out}}_{\mathrm{sphere}}$ can be calculated analytically. The situation is more complicated for the Bruggeman formalism, because both the inclusion and the host media are supposedly dispersed as particulates in the HCM itself. Except for the simple cases of isotropic dielectric/magnetic composites and impedance-matched chiral composites [21], Eq. (10) must be solved numerically for $\underline{\underline{C}}^{\mathrm{Br}}$.

Numerical implementation of the Br formalism is arduous for two reasons:

1. Equation (10) comprises 36 coupled nonlinear equations for the scalar components of $\underline{\underline{C}}^{\mathrm{Br}}$, and iterative techniques have to be used.

2. As both $\underline{\underline{D}}^{\text{in/Br}}$ and $\underline{\underline{D}}^{\text{out/Br}}$ are functions of the unknown $\underline{\underline{C}}^{\text{Br}}$, they have to be recalculated in each iteration. In general, this requires the evaluation of several two-dimensional numerical integrals per iteration.

In order to use the very simple Jacobi iterative technique [19], Eq. (10) is rewritten as

$$\underline{\underline{C}}^{\text{Br}} = T(\underline{\underline{C}}^{\text{Br}}), \tag{13}$$

where the operator T is defined as

$$\begin{aligned} T(\underline{\underline{C}}^{\text{Br}}) = \Big\{ & f\underline{\underline{C}}^{\text{in}} \cdot \left[\underline{\underline{I}} + i\omega \cdot \underline{\underline{D}}^{\text{in/Br}} \cdot (\underline{\underline{C}}^{\text{in}} - \underline{\underline{C}}^{\text{Br}})\right]^{-1} \\ & +(1-f)\underline{\underline{C}}^{\text{out}} \cdot \left[\underline{\underline{I}} + i\omega \underline{\underline{D}}^{\text{out/Br}} \cdot (\underline{\underline{C}}^{\text{out}} - \underline{\underline{C}}^{\text{Br}})\right]^{-1} \Big\} \\ & \cdot \Big\{ f\left[\underline{\underline{I}} + i\omega \underline{\underline{D}}^{\text{in/Br}} \cdot (\underline{\underline{C}}^{\text{in}} - \underline{\underline{C}}^{\text{Br}})\right]^{-1} \\ & +(1-f)\left[\underline{\underline{I}} + i\omega \underline{\underline{D}}^{\text{out/Br}} \cdot (\underline{\underline{C}}^{\text{out}} - \underline{\underline{C}}^{\text{Br}})\right]^{-1} \Big\}^{-1}. \end{aligned} \tag{14}$$

Equation (13) is iteratively solved as

$$\underline{\underline{C}}^{\text{Br}}[n] = T(\underline{\underline{C}}^{\text{Br}}[n-1]) \quad (n = 1, 2, \ldots), \tag{15}$$

with the starting value $\underline{\underline{C}}^{\text{Br}}[0] = \underline{\underline{C}}^{\text{MG}}$. If the ∞-norm of the Jacobian matrix of the 6×6 matrix operator T is less than unity, the iterative scheme converges to a fixed point [19], which is the required solution.

Calculations were performed on a DEC-ALPHA 3000-500 (150 MHz) workstation. For composites with uniaxial symmetric parts of the constitutive properties – discussed in the previous section – the depolarization dyadics were calculated analytically, and approximately 0.002 CPU seconds were needed to evaluate $\underline{\underline{C}}^{\text{MG}}$. In all other cases, a two-dimensional integration was performed to determine various depolarization dyadics. We used the one-dimensional adaptive integration technique described by Press et al. [23] for each of the two dimensions. The quality of the results was checked by comparing them with the analytical solution for the composites with uniaxial symmetric parts. In all cases, the relative error was below 10^{-6}. The implementation of the MG formalism using the adaptive integration technique needed $\leqslant 2$ CPU seconds. Also, we found that our iterative scheme for the Br formalism converged rapidly in most cases. Typically, 6 or fewer iterations were needed to obtain $\underline{\underline{C}}^{\text{Br}}$ with a relative error of less than 10^{-4}. The computation time for the Br formalism is about $N + 1$ times that for the MG formalism, where N is the number of iterations.

4. Numerical results

As mentioned earlier, we focus in this section on simple yet *non-trivial* applications of the MG and the Br formalisms. In particular, we do not discuss the homogenization of isotropic dielectric/magnetic and chiral composite media [4,5]. Furthermore, we exclude uniaxial dielectric composites which have been thoroughly discussed recently elsewhere [20,22], and against which we tested our computer code successfully.

4.1. Example 1

Let us consider non-dissipative spherical chiral inclusions embedded in a dissipative uniaxial dielectric host medium; thus,

$$\underline{\underline{\epsilon}}^{\text{in}} = \epsilon_0 \epsilon_r^{\text{in}} \underline{\underline{I}}, \qquad \underline{\underline{\mu}}^{\text{in}} = \mu_0 \mu_r^{\text{in}} \underline{\underline{I}}, \qquad \underline{\underline{\zeta}}^{\text{in}} = -\underline{\underline{\xi}}^{\text{in}} = \mathrm{i}\sqrt{\mu_0\epsilon_0}\, \kappa^{\text{in}} \underline{\underline{I}}, \tag{16}$$

for the inclusion medium, and

$$\underline{\underline{\epsilon}}^{\text{out}} = \epsilon_0 \left(\epsilon_t^{\text{out}} \underline{\underline{I}}_t + \epsilon_c^{\text{out}} \underline{c}\,\underline{c} \right), \qquad \underline{\underline{\mu}}^{\text{out}} = \mu_0 \underline{\underline{I}}, \qquad \underline{\underline{\zeta}}^{\text{out}} = \underline{\underline{\xi}}^{\text{out}} = \underline{\underline{0}}, \tag{17}$$

for the host medium. Here, ϵ_0 and μ_0 are the permittivity and the permeability of free space (i.e., vacuum), $\underline{c}$ is a unit vector parallel to the crystallographic axis of the host medium, and $\underline{\underline{I}}_t = \underline{\underline{I}} - \underline{c}\,\underline{c}$. The following numerical values were chosen for calculations:

$$\epsilon_t^{\text{out}} = 1.5 + 0.1\mathrm{i}, \qquad \epsilon_c^{\text{out}} = 5 + \mathrm{i}, \tag{18}$$

$$\epsilon_r^{\text{in}} = 2, \qquad \mu_r^{\text{in}} = 1.5, \qquad \kappa^{\text{in}} = 1. \tag{19}$$

The HCM has to be uniaxial bianisotropic with

$$\underline{\underline{\epsilon}}^{\text{HCM}} = \epsilon_0 \left(\epsilon_t^{\text{HCM}} \underline{\underline{I}}_t + \epsilon_c^{\text{HCM}} \underline{c}\,\underline{c} \right), \tag{20}$$

$$\underline{\underline{\zeta}}^{\text{HCM}} = -\underline{\underline{\xi}}^{\text{HCM}} = \mathrm{i}\sqrt{\epsilon_0\mu_0} \left(\kappa_t^{\text{HCM}} \underline{\underline{I}}_t + \kappa_c^{\text{HCM}} \underline{c}\,\underline{c} \right), \tag{21}$$

$$\underline{\underline{\mu}}^{\text{HCM}} = \mu_0 \left(\mu_t^{\text{HCM}} \underline{\underline{I}}_t + \mu_c^{\text{HCM}} \underline{c}\,\underline{c} \right). \tag{22}$$

In Fig. 1(a)–(f), the MG and the Br estimates of the real and the imaginary parts of the constitutive parameters ϵ_t^{HCM}, ϵ_c^{HCM}, κ_t^{HCM}, κ_c^{HCM}, μ_t^{HCM}, μ_c^{HCM} are plotted as functions of f. These estimates clearly demonstrate how bianisotropy is created by mixing isotropic chiral and anisotropic dielectric media. We also observe from Fig. 1(c) and (f) that – except for ϵ_t^{HCM} and ϵ_c^{HCM} – the imaginary parts of the constitutive parameters of the HCM are predicted quite differently by the MG and the Br formalisms.

4.2. Example 2

Next we homogenize spheroidal chiral inclusions in an isotropic host medium. Whereas bianisotropy was generated by the uniaxial dielectric host medium in Example 1, it is now caused by the non-spherical shape of the inclusions. For calculations, we chose the host medium to be free space ($\underline{\underline{\epsilon}}^{\text{out}} = \epsilon_0 \underline{\underline{I}}$, $\underline{\underline{\mu}}^{\text{out}} = \mu_0 \underline{\underline{I}}$ and $\underline{\underline{\zeta}}^{\text{out}} = \underline{\underline{\xi}}^{\text{out}} = \underline{\underline{0}}$), while the inclusion medium is described by Eqs (16) and (19). We chose prolate spheroidal inclusions with an aspect ratio of 5. As in Example 1, the HCM is uniaxial bianisotropic.

The MG formalism is trivial in this example, because the well-known free-space depolarization dyadics for spherical and spheroidal exclusion regions [7,18] are used in Eqs (3) and (9). The Br formalism, in contrast, requires the evaluation of the depolarization dyadic of the uniaxial bianisotropic HCM. In order to avoid additional free parameters [20], the exclusion regions for evaluating $\underline{\underline{\mathbf{D}}}^{\text{in/Br}}$ and $\underline{\underline{\mathbf{D}}}^{\text{out/Br}}$ were chosen to be identical in shape with the inclusions.

Estimates of the constitutive parameters ϵ_t^{HCM}, ϵ_c^{HCM}, κ_t^{HCM}, κ_c^{HCM}, μ_t^{HCM} and μ_c^{HCM} are plotted in Fig. 2(a)–(c) as functions of the inclusion volume concentration f. The transverse constitutive

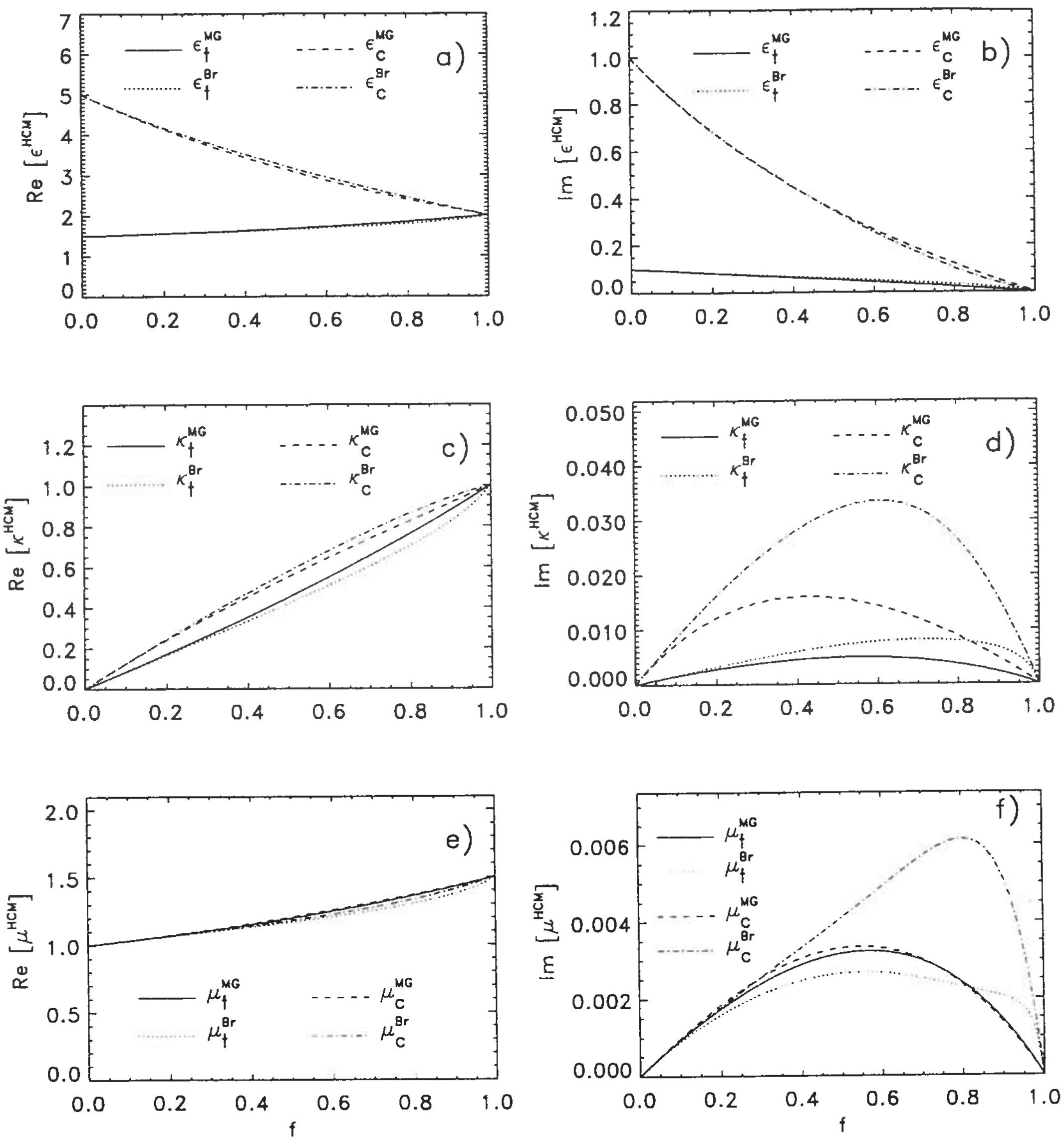

Fig. 1. Bruggeman (Br) and Maxwell Garnett (MG) estimates of the constitutive parameters ε_t^{HCM}, ε_c^{HCM}, κ_t^{HCM}, κ_c^{HCM}, μ_t^{HCM} and μ_c^{HCM} of a HCM comprising spherical chiral inclusions in a uniaxial dielectric host medium. The inclusion volume concentration f varies from 0 to 1, but the MG estimates are not expected to hold for $f > 0.3$. The input parameters are $\varepsilon_r^{in} = 2$, $\mu_r^{in} = 1.5$, $\kappa^{in} = 1$, $\varepsilon_t^{out} = 1.5 + 0.1i$ and $\varepsilon_c^{out} = 5 + i$.

parameters ε_t^{HCM}, κ_t^{HCM} and μ_t^{HCM} turned out to be rather insensitive to the homogenization formalism used. This is not true in the axial direction, there being pronounced differences between the MG and the Br estimates of ε_c^{HCM}, κ_c^{HCM} and μ_c^{HCM}. Parenthetically, the simple Jacobi iteration scheme showed a poor convergence for f close to unity. For $f > 0.95$, we did not reach convergence even after 200 iteration cycles, but we did not pursue this somewhat extreme case any further. However, several ways are available to overcome convergence problems, e.g., by seeking an alternative to Eq. (14) or by using relaxation methods [19,23].

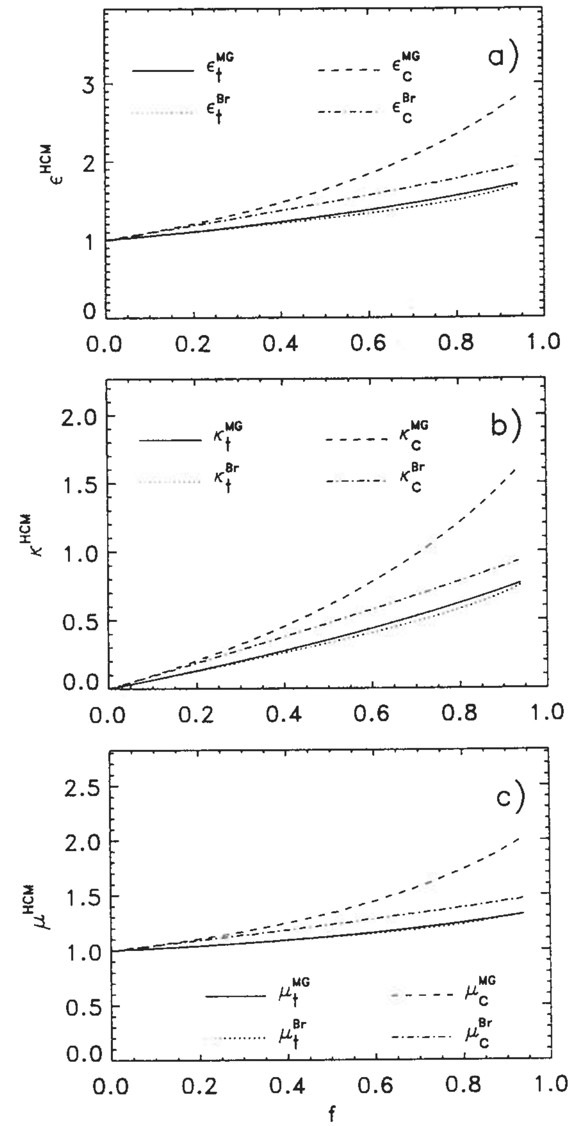

Fig. 2. Same as Fig. 1, but for prolate spheroidal chiral inclusions with an aspect ratio of 5 : 1, embedded in free space. The input parameters for the inclusion medium are the same as in Fig. 1.

4.3. *Example 3*

This example is an extension of Example 1 to a biaxial dielectric host medium, with

$$\underline{\underline{\varepsilon}}^{\text{out}} = \varepsilon_0\left(\varepsilon_1^{\text{out}}\,\underline{c}_1\,\underline{c}_1 + \varepsilon_2^{\text{out}}\,\underline{c}_2\,\underline{c}_2 + \varepsilon_3^{\text{out}}\,\underline{c}_3\,\underline{c}_3\right), \tag{23}$$

the unit vectors $\underline{c}_1, \underline{c}_2$ and $\underline{c}_3$ being mutually orthogonal. Two-dimensional integrations must now be carried out to obtain the depolarization dyadics. In order to keep the parameter space manageable, we assumed the host medium to be non-dissipative in our calculations: $\varepsilon_1^{\text{out}} = 5$, $\varepsilon_2^{\text{out}} = 2$ and $\varepsilon_3^{\text{out}} = 1$. The inclusions are the same as in Example 1, with numerical values given in Eq. (19).

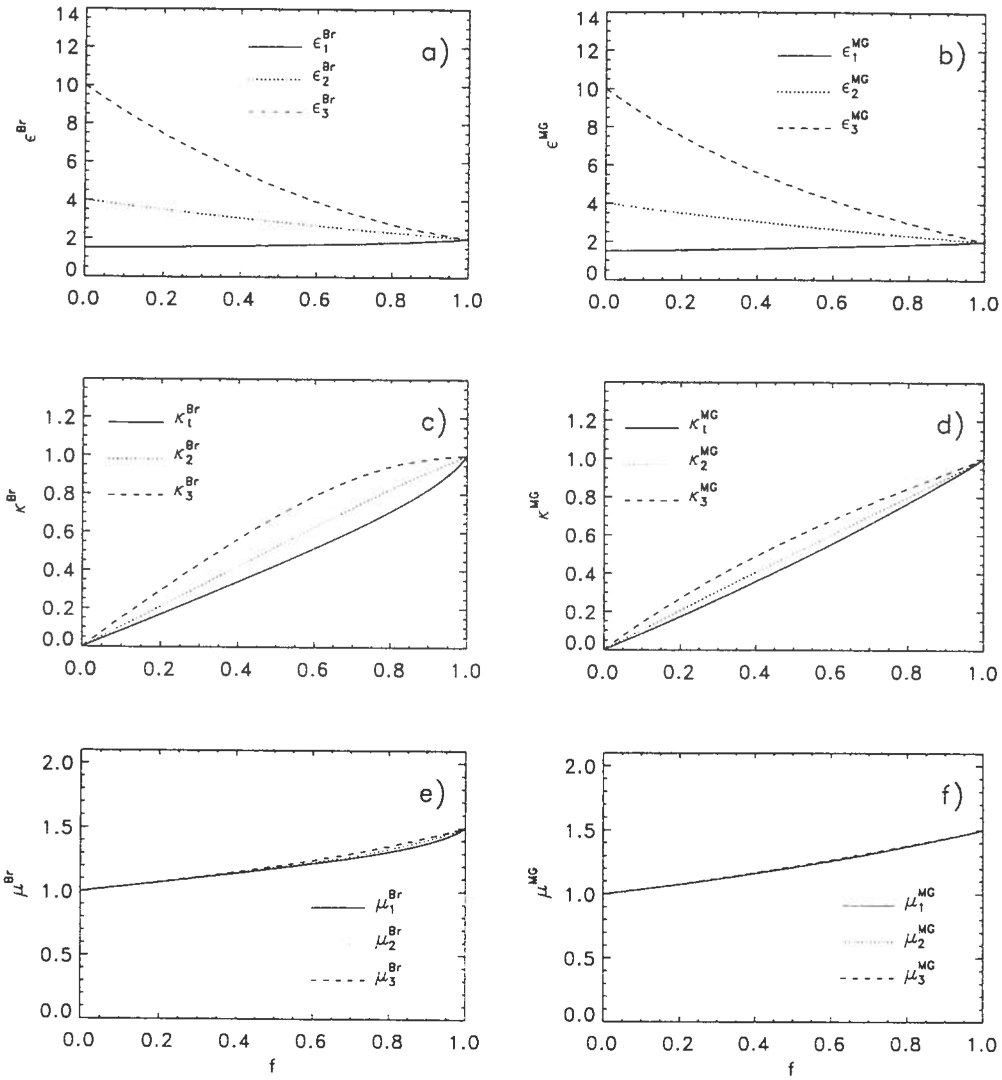

Fig. 3. Same as Fig. 1, but for a biaxial lossless dielectric host medium with $\varepsilon_1^{\rm out} = 5$, $\varepsilon_2^{\rm out} = 2$ and $\varepsilon_3^{\rm out} = 1$. Figures (a), (c), (e) show the Br estimate, while (b), (d), (f) show the corresponding MG estimate.

The HCM is biaxial bianisotropic; thus,

$$\underline{\underline{\varepsilon}}^{\rm HCM} = \varepsilon_0 \sum_{j=1}^{3} \varepsilon_j^{\rm HCM} \, \underline{c}_j \underline{c}_j \,, \tag{24}$$

$$\underline{\underline{\zeta}}^{\rm HCM} = -\underline{\underline{\xi}}^{\rm HCM} = {\rm i}\sqrt{\varepsilon_0 \mu_0} \sum_{j=1}^{3} \kappa_j^{\rm HCM} \, \underline{c}_j \underline{c}_j \,, \tag{25}$$

$$\underline{\underline{\mu}}^{\rm HCM} = \mu_0 \sum_{j=1}^{3} \mu_j^{\rm HCM} \, \underline{c}_j \underline{c}_j \,. \tag{26}$$

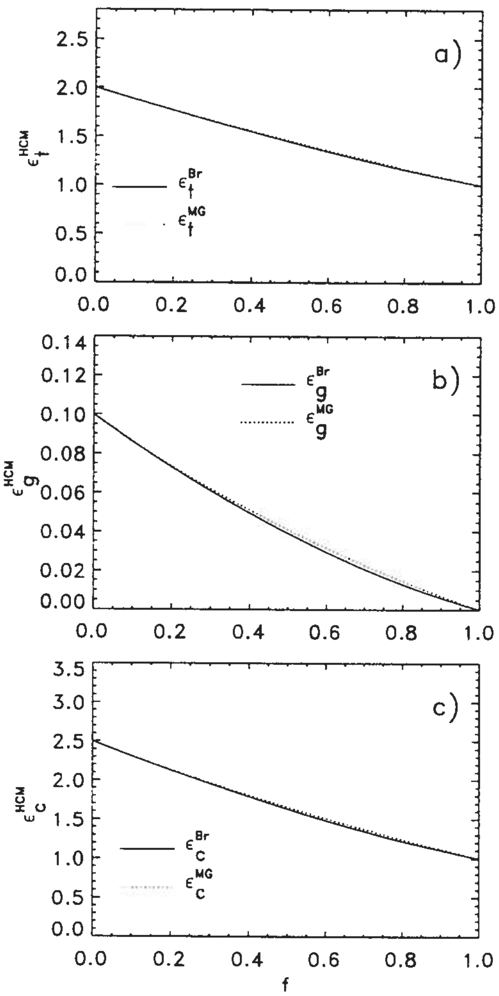

Fig. 4. Br and MG estimates of ε_t^{HCM}, ε_g^{HCM} and ε_c^{HCM} of a HCM comprising spherical voids in a gyrotropic dielectric host medium. The input parameters are $\varepsilon_t^{out} = 2$, $\varepsilon_c^{out} = 2.5$ and $\varepsilon_g^{out} = 0.1$. The MG estimate is not expected to apply for $f > 0.3$.

The Br estimates ε_j^{Br}, κ_j^{Br}, and μ_j^{Br} $(j = 1, 2, 3)$ are plotted in Fig. 3(a), (c) and (e) as functions of f. The corresponding MG estimates are shown in Fig. 3(b), (d) and (f). Clearly, the biaxiality of κ_j^{HCM} and μ_j^{HCM} stems from the biaxiality of $\underline{\underline{\varepsilon}}^{out}$. These figures also show that ε_j^{HCM}, μ_j^{HCM} and κ_j^{HCM} do not have any common crystallographic axes [24].

4.4. Example 4

For the last example, let the host medium be gyrotropic dielectric with

$$\underline{\underline{\varepsilon}}^{out} = \varepsilon_0 \left[\varepsilon_t^{out} \underline{\underline{I}} + (\varepsilon_t^{out} - \varepsilon_c^{out}) \underline{c}\,\underline{c} - i\varepsilon_g^{out} \underline{c} \times \underline{\underline{I}} \right], \tag{27}$$

where ε_t^{out}, ε_c^{out} and ε_g^{out} are scalar relative permittivities and $\underline{c}$ is a unit vector parallel to the biasing quasistatic magnetic field. The inclusions are spherical voids, and $\varepsilon_t^{out} = 2$, $\varepsilon_c^{out} = 2.5$ and $\varepsilon_g^{out} = 0.1$ were set for numerical work.

The HCM is a gyrotropic dielectric with

$$\varepsilon^{\mathrm{HCM}} = \varepsilon_0 \left[\varepsilon_{\mathrm{t}}^{\mathrm{HCM}} \underline{\underline{I}} + (\varepsilon_{\mathrm{t}}^{\mathrm{HCM}} - \varepsilon_{\mathrm{c}}^{\mathrm{HCM}}) \underline{c}\,\underline{c} - \mathrm{i}\varepsilon_{g}^{\mathrm{HCM}}\, \underline{c} \times \underline{\underline{I}}\right], \tag{28}$$

as its permittivity dyadic. The MG and the Br estimates for $\varepsilon_{\mathrm{t}}^{\mathrm{HCM}}$, $\varepsilon_{\mathrm{c}}^{\mathrm{HCM}}$ and $\varepsilon_{\mathrm{g}}^{\mathrm{HCM}}$ are plotted in Fig. 4(a)–(c) as functions of f. Both estimates are virtually identical, except for the gyrotropic part $\varepsilon_{\mathrm{g}}^{\mathrm{HCM}}$, for this Swiss cheese composite medium.

5. Concluding remarks

In this paper, we presented numerical studies on the homogenization of linear bianisotropic particulate composites with ellipsoidal inclusions. The main goal of the paper was to demonstrate the feasibility of numerical calculations as well as the availablility of a computer program suitable for this purpose.

Although our computer program is applicable to a very general case, we restricted the presented results to composites having certain symmetries for simplicity. In two of the examples (Examples 1 and 2), we found pronounced differences between the MG and the Br estimates. This clearly shows the need to have both formalisms at hand, for instance, when interpreting experimental results.

We note that the general case of bianisotropic-in-biansotropic composites is of great practical importance. For instance, artificial isotropic chiral composites are too difficult to be fabricated, the method of fabrication often inducing anisotropy [25]. This anisotropy can be accounted for in our formalisms, thereby allowing more accurate description of the electromagnetic response properties of the chiral composites.

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Erratum

Homogenization of linear bianisotropic particulate composite media – Numerical studies
by B. Michel, A. Lakhtakia and W.S. Weiglhofer

[*International Journal of Applied Electromagnetics and Mechanics* 9(2) (1998), 167–178]

Two errors appeared in Section 4 of this paper.

- First, due to a programming error, the plots of the Bruggeman (Br) estimates in Fig. 1 are incorrect. The corrected Fig. 1 is reproduced here (see next page).
- Second, the constitutive parameters of the dielectric host medium for Example 3, Section 4.3, were chosen as $\varepsilon_1^{\rm out} = 1.5$, $\varepsilon_2^{\rm out} = 4$, $\varepsilon_3^{\rm out} = 10$, and not as given in the text below Eq. (23) and the caption of Fig. 3.

Neither of these errors alters our analytical technique, solution algorithm and conclusions.

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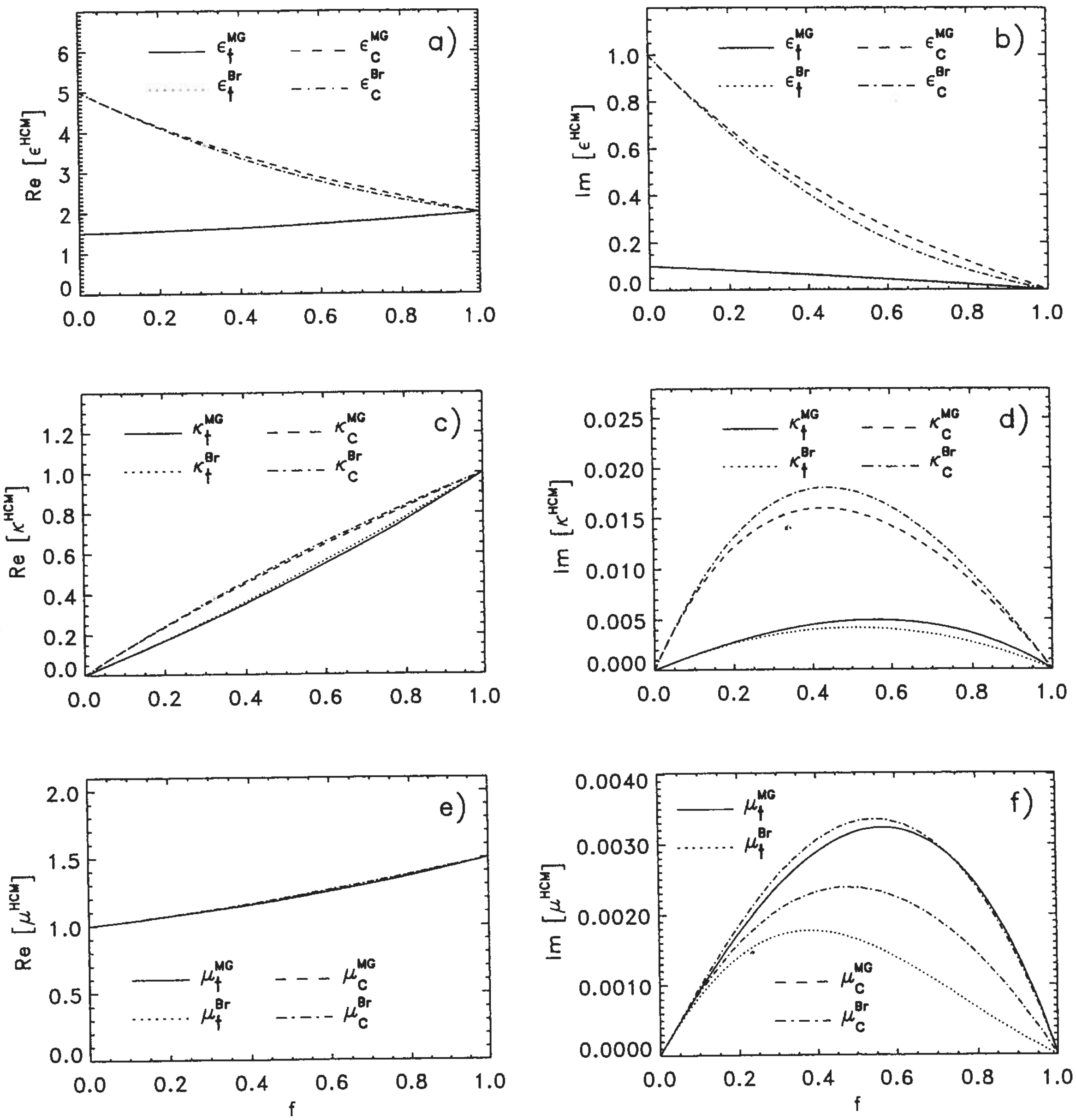

Fig. 1. Bruggeman (Br) and Maxwell Garnett (MG) estimates of the constitutive parameters ε_t^{HCM}, ε_c^{HCM}, κ_t^{HCM}, κ_c^{HCM}, μ_t^{HCM} and μ_c^{HCM} of a HCM comprising spherical chiral inclusions in a uniaxial dielectric host medium. The inclusion volume concentration f varies from 0 to 1, but the MG estimates are not expected to hold for $f > 0.3$. The input parameters are $\varepsilon_r^{in} = 2$, $\mu_r^{in} = 1.5$, $\kappa^{in} = 1$, $\varepsilon_t^{out} = 1.5 + 0.1i$ and $\varepsilon_c^{out} = 5 + i$.