Time-Harmonic Dyadic Green's Functions for Reflection and Transmission by a Chiral Slab

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The canonical problem of time-harmonic radiation from sources in the presence of a chiral slab has been developed by formulating the relevant dyadic Green's functions; this problem is of interest for the purpose of electromagnetic shielding.

Zeitharmonische dyadische Greensche Funktionen für Reflexion und Übertragung bei einer chiralen Platte

Das kanonische Problem der zeitharmonischen Abstrahlung von Quellen bei einer chiralen Platte wurde untersucht und führte zur Formulierung der entsprechenden dyadischen Greenschen Funktionen. Das Problem ist von Interesse bei der elektromagnetischen Schirmung.

1. Introduction

Attention is increasingly being paid to canonical problems regarding the behavior of electromagnetic fields in isotropic chiral media [1], [2] as it has become apparent that these materials have potentially exciting applications as coating materials [3], [4], for control of the polarization characteristics of antennas [5] and lenses [6], for RCS alteration [7], among others.

In particular, the major utility of these materials is being envisioned nowadays in terms of artificial radar-absorbing strata [2], [8], [9] that owe inspiration to a 1914 experiment of Lindman [10], and provides the motivation for the developments in this paper. The principal results obtained are the (time-harmonic) dyadic Green's functions that characterise the reflection and transmission properties of a chiral slab of finite thickness and infinite extent. The special case of a vertical electric dipole radiating in the presence of a chiral slab has then been given for the purpose of illustration.

2. Planewave Response of a Chiral Slab

The geometry of the problem being considered is as follows: The lower halfspace $z \leq 0$ as well as the zone $z \geq d$ are occupied by free space ($\boldsymbol{D} = \varepsilon_0 \boldsymbol{E}$, $\boldsymbol{B} = \mu_0 \boldsymbol{H}$), while the infinite slab $0 \leq z \leq d$ is occupied by an isotropic chiral medium ($\boldsymbol{D} = \varepsilon \boldsymbol{E} + \varepsilon\beta \nabla \times \boldsymbol{E}$, $\boldsymbol{B} = \mu \boldsymbol{H} + \mu\beta \nabla \times \boldsymbol{H}$). Using an $\exp(-\mathrm{i}\omega t)$ time-dependence, the following quantities are defined: $k_0 = \omega\sqrt{\mu_0 \varepsilon_0}$ and $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$ for free space; $k = \omega\sqrt{\mu\varepsilon}$, $\eta = \sqrt{\mu/\varepsilon}$, $\gamma_1 = k/(1 - k\beta)$ and $\gamma_2 = k/(1 + k\beta)$ for the chiral medium. It is understood that all quantities thus defined have finite magnitudes; furthermore, $\mathrm{Re}[\eta] > 0$, $\mathrm{Re}[\gamma_1] > 0$, $\mathrm{Re}[\gamma_2] > 0$, $\mathrm{Im}[\gamma_1] \geq 0$ and $\mathrm{Im}[\gamma_2] \geq 0$.

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Without loss of generality, we take the source to lie in the lower halfspace $z \leq 0$ for the reflection-transmission problem. Furthermore, it is known that the free-space dyadic Green's function can be decomposed into a spectrum of plane waves [11]. Therefore, we begin by considering the reflection and transmission of a plane wave by the chiral slab.

Let the cartesian unit vectors be denoted by $\boldsymbol{u}_x$, $\boldsymbol{u}_y$ and $\boldsymbol{u}_z$. After defining the quantities

$$\begin{aligned}
\boldsymbol{k}_{0\pm}\{\varkappa_x, \varkappa_y\} &= \varkappa_x \boldsymbol{u}_x + \varkappa_y \boldsymbol{u}_y \pm k_{0z} \boldsymbol{u}_z,\\
\boldsymbol{t}_{\pm}\{\varkappa_x, \varkappa_y\} &= -(1/\varkappa)(\varkappa_y \boldsymbol{u}_x - \varkappa_x \boldsymbol{u}_y),\\
\boldsymbol{p}_{0\pm}\{\varkappa_x, \varkappa_y\} &= -(\pm k_{0z}/\varkappa k_0)(\varkappa_x \boldsymbol{u}_x + \varkappa_y \boldsymbol{u}_y) + (\varkappa/k_0)\boldsymbol{u}_z,\\
\varkappa\{\varkappa_x, \varkappa_y\} &= +\sqrt{\varkappa_x^2 + \varkappa_y^2},\\
k_{0z}\{\varkappa_x, \varkappa_y\} &= +\sqrt{k_0^2 - \varkappa^2},
\end{aligned}$$

that are functions of the horizontal wavenumbers $\varkappa_x$ and $\varkappa_y$, we consider that in the lower halfspace [12], [13],

$$\begin{aligned}
\boldsymbol{E}(\boldsymbol{r}) = [\{A_\perp \boldsymbol{t}_+ + A_\parallel \boldsymbol{p}_{0+}\} &\cdot \\
\cdot \exp(\mathrm{i}k_{0z}z) + \{B_\perp \boldsymbol{t}_- + B_\parallel \boldsymbol{p}_{0-}\} &\cdot \\
\cdot \exp(-\mathrm{i}k_{0z}z)] \exp(\mathrm{i}\varkappa_x x + \mathrm{i}\varkappa_y y), &\quad z \leq 0,
\end{aligned} \tag{1a}$$

and

$$\boldsymbol{H}(\boldsymbol{r}) = \nabla \times \boldsymbol{E}(\boldsymbol{r})/\mathrm{i}\omega\mu_0, \quad z \leq 0. \tag{1b}$$

It is clear that the coefficients $\{A_\perp, A_\parallel\}$ represent a plane wave that is incident on the slab from the lower halfspace, while the coefficients $\{B_\perp, B_\parallel\}$ represent the plane wave consequently reflected into the same halfspace. Similarly, the plane wave transmitted into the upper space $z \geq d$ is set down as

$$\begin{aligned}
\boldsymbol{E}(\boldsymbol{r}) = [\{C_\perp \boldsymbol{t}_+ + C_\parallel \boldsymbol{p}_{0+}\} \exp(\mathrm{i}k_{0z}z)] &\cdot \\
\cdot \exp(\mathrm{i}\varkappa_x x + \mathrm{i}\varkappa_y y), &\quad z \geq d
\end{aligned} \tag{2a}$$

$$\boldsymbol{H}(\boldsymbol{r}) = \nabla \times \boldsymbol{E}(\boldsymbol{r})/\mathrm{i}\omega\mu_0, \quad z \geq d, \tag{2b}$$

with transmission coefficients $\{C_\perp, C_\parallel\}$.

In the chiral slab, the field representation is somewhat more complicated; thus [1], [2], [14],

$$\begin{aligned}
\boldsymbol{E}(\boldsymbol{r}) &= \boldsymbol{Q}_1(\boldsymbol{r}) - \mathrm{i}\eta \boldsymbol{Q}_2(\boldsymbol{r}),\\
\boldsymbol{H}(\boldsymbol{r}) &= -\mathrm{i}\boldsymbol{Q}_1(\boldsymbol{r})/\eta + \boldsymbol{Q}_2(\boldsymbol{r}), \quad 0 \leq z \leq d,
\end{aligned} \tag{3}$$

where

$$Q_1(r) = [A_1(t_- + ip_{1-}\} \exp(-i\gamma_{1z} z) + B_1(t_+ + ip_{1+}\} \exp(i\gamma_{1z} z)] \exp(i\varkappa_x x + i\varkappa_y y), \quad 0 \le z \le d, \quad (4a)$$

$$Q_2(r) = [A_2(t_- - ip_{2-}\} \exp(-i\gamma_{2z} z) + B_1(t_+ - ip_{2+}\} \exp(i\gamma_{2z} z)] \exp(i\varkappa_x x + i\varkappa_y y), \quad 0 \le z \le d. \quad (4b)$$

Here,

$$p_{1\pm}\{\varkappa_x, \varkappa_y\} = -(\pm\gamma_{1z}/\varkappa\gamma_1)(\varkappa_x u_x + \varkappa_y u_y) + (\varkappa/\gamma_1) u_z,$$
$$p_{2\pm}\{\varkappa_x, \varkappa_y\} = -(\pm\gamma_{2z}/\varkappa\gamma_2)(\varkappa_x u_x + \varkappa_y u_y) + (\varkappa/\gamma_2) u_z,$$
$$\gamma_{1z}\{\varkappa_x, \varkappa_y\} = +\sqrt{\gamma_1^2 - \varkappa^2},$$
$$\gamma_{2z}\{\varkappa_x, \varkappa_y\} = +\sqrt{\gamma_2^2 - \varkappa^2},$$

are also functions of the horizontal wavenumbers $\varkappa_x$ and $\varkappa_y$, and are consistent with Snel's laws.

Following [15], continuity of the tangential E and the tangential H fields at the interface $z=0$ leads to the matrix relations,

$$\begin{pmatrix} B_\perp \\ B_\parallel \end{pmatrix} = [r] \begin{pmatrix} A_\perp \\ A_\parallel \end{pmatrix} + [T] \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}, \quad (5a)$$

$$\begin{pmatrix} B_1 \\ B_2 \end{pmatrix} = [R] \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} + [t] \begin{pmatrix} A_\perp \\ A_\parallel \end{pmatrix}, \quad (5b)$$

where the 2×2 matrices $[r]$, $[t]$, $[R]$ and $[T]$ are to be interpreted as

$$[r] = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix},$$

etc. The Fresnel coefficients utilized in (5a, b) are not functions of $\varkappa_x$ and $\varkappa_y$ independently, but of $\varkappa^2$ and are given as [15]

$$D = (\eta_0^2 + \eta^2)(\xi_1 + \xi_2) + 2\eta_0 \eta (\xi_1 \xi_2 + 1),$$
$$r_{11}\{\varkappa^2\} = -[(\eta_0^2 - \eta^2)(\xi_1 + \xi_2) + 2\eta_0 \eta(\xi_1 \xi_2 - 1)]/D,$$
$$r_{22}\{\varkappa^2\} = [(\eta_0^2 - \eta^2)(\xi_1 + \xi_2) - 2\eta_0 \eta(\xi_1 \xi_2 - 1)]/D,$$
$$r_{12}\{\varkappa^2\} = 2i\eta_0 \eta(\xi_1 - \xi_2)/D,$$
$$r_{21}\{\varkappa^2\} = -r_{12}\{\varkappa^2\},$$
$$t_{11}\{\varkappa^2\} = 2\eta(\eta \xi_2 + \eta_0)/D,$$
$$t_{22}\{\varkappa^2\} = -2(\eta_0 \xi_1 + \eta)/D,$$
$$t_{12}\{\varkappa^2\} = -2i\eta(\eta_0 \xi_2 + \eta)/D,$$
$$t_{21}\{\varkappa^2\} = 2i(\eta \xi_1 + \eta_0)/D,$$
$$R_{11}\{\varkappa^2\} = [(\eta_0^2 + \eta^2)(\xi_1 - \xi_2) + 2\eta_0 \eta(\xi_1 \xi_2 - 1)]/D,$$
$$R_{22}\{\varkappa^2\} = [-(\eta_0^2 + \eta^2)(\xi_1 - \xi_2) + 2\eta_0 \eta(\xi_1 \xi_2 - 1)]/D,$$
$$R_{12}\{\varkappa^2\} = -2i\eta \xi_2(\eta_0^2 - \eta^2)/D,$$
$$R_{21}\{\varkappa^2\} = 2i\xi_1(\eta_0^2 - \eta^2)/\eta D,$$
$$T_{11}\{\varkappa^2\} = 4\eta_0 \xi_1(\eta \xi_2 + \eta_0)/D,$$
$$T_{22}\{\varkappa^2\} = -4\eta \eta_0 \xi_2(\eta_0 \xi_1 + \eta)/D,$$
$$T_{12}\{\varkappa^2\} = -4i\eta \eta_0 \xi_2(\eta \xi_1 + \eta_0)/D,$$
$$T_{21}\{\varkappa^2\} = 4i\eta_0 \xi_1(\eta_0 \xi_2 + \eta)/D,$$

where

$$\xi_1\{\varkappa^2\} = k_0 \gamma_{1z}/k_{0z}\gamma_1, \quad \xi_2\{\varkappa^2\} = k_0 \gamma_{2z}/k_{0z}\gamma_2.$$

Similarly, matching of the tangential fields at $z=d$ leads to

$$\begin{pmatrix} C_\perp \\ C_\parallel \end{pmatrix} = \exp(-ik_{0z} d)[T][F] \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} \quad (5c)$$

and

$$\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = [F][R][F] \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}, \quad (5d)$$

with

$$[F] = \begin{bmatrix} \exp(i\gamma_{1z} d) & 0 \\ 0 & \exp(i\gamma_{2z} d) \end{bmatrix}.$$

Elimination of the coefficients A_1, A_2, B_1 and B_2 from (5a–d) yields the planewave reflection-transmission response of the slab in matrix form as

$$\begin{pmatrix} B_\perp \\ B_\parallel \end{pmatrix} = [\varrho] \begin{pmatrix} A_\perp \\ A_\parallel \end{pmatrix}, \quad (6a)$$

$$\begin{pmatrix} C_\perp \\ C_\parallel \end{pmatrix} = \exp(-ik_{0z} d)[\tau] \begin{pmatrix} A_\perp \\ A_\parallel \end{pmatrix}, \quad (6b)$$

where

$$[\varrho] = [r] + [T][F][R][F]\{[I] - [R][F][R][F]\}^{-1}[t], \quad (7a)$$
$$[\tau] = [T][F]\{[I] - [R][F][R][F]\}^{-1}[t]. \quad (7b)$$

The 2×2 matrices $[\varrho]$ and $[\tau]$ can be calculated for given values for $\varkappa_x$ and $\varkappa_y$, and will be utilized in the construction of the Green's dyadics for reflection and transmission by the chiral slab. Parenthetically, it is noted that conservation of energy guarantees that $|\varrho_{11}|^2 + |\varrho_{21}|^2 + |\tau_{11}|^2 + |\tau_{21}|^2 \le 1$ and $|\varrho_{12}|^2 + |\varrho_{22}|^2 + |\tau_{12}|^2 + |\tau_{22}|^2 \le 1$ if k_{0z} is purely real, the inequalities in these relations coming in only if the chiral slab is intrinsically lossy [1], [2]. It is also mentioned here that the $\varkappa$, for which the matrix $[\varrho]$ becomes singular, should be termed the Brewster wavenumber for the present boundary value problem, in the spirit of [15], [16].

3. Dyadic Green's Functions

Since the source as well as the field points lie exclusively in the lower halfspace here, the electric field $E(r)$ can be obtained from source current densities through

$$E(r) = i\omega\mu_0 \iiint d^3r' [\underline{G}_e(r, r') \cdot J(r')] - \iiint d^3r' [\underline{G}_m(r, r') \cdot K(r')], \quad z' < 0, \ z \le 0 \text{ or } z \ge d, \quad (8)$$

provided the dyadic Green's functions $\underline{G}_e(r, r')$ and $\underline{G}_m(r, r')$ are known; $J(r')$ and $K(r')$, respectively, are the source electric and magnetic current densities; r is the field point and r' is the source point.

Synthesis of these dyadics in the lower halfspace can be accomplished through the decompositions

$$\underline{G}_e(r, r') = \underline{G}_{e0}(r, r') + \underline{G}_{e\varrho}(r, r'), \quad z \le 0, \ z' < 0 \quad (9a)$$
$$\underline{G}_m(r, r') = \underline{G}_{m0}(r, r') + \underline{G}_{m\varrho}(r, r'), \quad z \le 0, \ z' < 0 \quad (9b)$$

where *only* $\underline{G}_{e\varrho}$ and $\underline{G}_{m\varrho}$ take into account the presence of the chiral slab. The dyadics $\underline{G}_{e0}$ and $\underline{G}_{m0}$ are the usual free space Green's dyadics,

$$\underline{G}_{e0}(\boldsymbol{r},\boldsymbol{r}') = \{\underline{I} + \nabla\nabla/k_0^2\}\,[\exp(\mathrm{i}\,k_0 R)/4\pi R], \quad (10\mathrm{a})$$

$$\underline{G}_{m0}(\boldsymbol{r},\boldsymbol{r}') = \{\nabla \times \underline{I}\}\,[\exp(\mathrm{i}\,k_0 R)/4\pi R], \quad (10\mathrm{b})$$

and yield the solution of Maxwell's equations if the entire space ($|z| \geq 0$) is vacuous; $\boldsymbol{R} = \boldsymbol{r} - \boldsymbol{r}'$.

The decompositions (9a, b) naturally lead to a similar decomposition of the electromagnetic field in the lower halfspace; thus,

$$\boldsymbol{E}(\boldsymbol{r}) = \boldsymbol{E}_0(\boldsymbol{r}) + \boldsymbol{E}_\varrho(\boldsymbol{r}), \quad z \leq 0, \quad (11)$$

where

$$\boldsymbol{E}_0(\boldsymbol{r}) = \mathrm{i}\omega\mu_0 \iiint \mathrm{d}^3 r' [\underline{G}_{e0}(\boldsymbol{r},\boldsymbol{r}') \cdot \boldsymbol{J}(\boldsymbol{r}')] - \iiint \mathrm{d}^3 r' [\underline{G}_{m0}(\boldsymbol{r},\boldsymbol{r}') \cdot \boldsymbol{K}(\boldsymbol{r}')], \quad (12\mathrm{a})$$

is the primary field that is radiated by the sources into free space without taking any interfaces into consideration. In turn, this primary field is incidence on the slab, thereby giving rise to the reflected field

$$\boldsymbol{E}_\varrho(\boldsymbol{r}) = \mathrm{i}\omega\mu_0 \iiint \mathrm{d}^3 r' [\underline{G}_{e\varrho}(\boldsymbol{r},\boldsymbol{r}') \cdot \boldsymbol{J}(\boldsymbol{r}')] - \iiint \mathrm{d}^3 r' [\underline{G}_{m\varrho}(\boldsymbol{r},\boldsymbol{r}') \cdot \boldsymbol{K}(\boldsymbol{r}')], \quad z<0. \quad (12\mathrm{b})$$

Now, the right hand sides of (10a) and (10b) can be set down as [11]

$$\underline{G}_{e0}(\boldsymbol{r},\boldsymbol{r}') = -\boldsymbol{u}_z\boldsymbol{u}_z\,\delta(\boldsymbol{R}) + (\mathrm{i}/8\pi^2)\int_{-\infty}^{\infty}\mathrm{d}\varkappa_x \int_{-\infty}^{\infty}\mathrm{d}\varkappa_y \cdot [(1/k_{0z})[\boldsymbol{t}_\pm \boldsymbol{t}_\pm + \boldsymbol{p}_{0\pm}\boldsymbol{p}_{0\pm}]\exp(\mathrm{i}\,\boldsymbol{k}_{0\pm}\cdot\boldsymbol{R})], \quad (13\mathrm{a})$$

and

$$\underline{G}_{m0}(\boldsymbol{r},\boldsymbol{r}') = -(k_0/8\pi^2)\int_{-\infty}^{\infty}\mathrm{d}\varkappa_x \int_{-\infty}^{\infty}\mathrm{d}\varkappa_y \cdot [(1/k_{0z})[\boldsymbol{p}_{0\pm}\boldsymbol{t}_\pm - \boldsymbol{t}_\pm\boldsymbol{p}_{0\pm}]\exp(\mathrm{i}\,\boldsymbol{k}_{0\pm}\cdot\boldsymbol{R})], \quad (13\mathrm{b})$$

respectively, with the upper (resp. lower) sign is to be taken for $z > z'$ (resp. $z < z'$); $\delta(\boldsymbol{R})$ is the Kronecker delta. Thus, the field incident on the slab is a continuous spectrum of plane waves; hence, the reflected field has also to be a continuous spectrum of plane waves [17]. Consequently, one can use the developments of the previous section to write down

$$\underline{G}_{e\varrho}(\boldsymbol{r},\boldsymbol{r}') = (\mathrm{i}/8\pi^2)\int_{-\infty}^{\infty}\mathrm{d}\varkappa_x \int_{-\infty}^{\infty}\mathrm{d}\varkappa_y [(1/k_{0z})\underline{R}_e\{\varkappa_x,\varkappa_y\} \cdot \exp(\mathrm{i}\,\boldsymbol{k}_{0-}\cdot\boldsymbol{r})\exp(-\mathrm{i}\,\boldsymbol{k}_{0+}\cdot\boldsymbol{r}')], \quad z \leq 0, z' < 0, \quad (14\mathrm{a})$$

$$\underline{G}_{m\varrho}(\boldsymbol{r},\boldsymbol{r}') = -(k_0/8\pi^2)\int_{-\infty}^{\infty}\mathrm{d}\varkappa_x \int_{-\infty}^{\infty}\mathrm{d}\varkappa_y [(1/k_{0z})\underline{R}_m\{\varkappa_x,\varkappa_y\} \cdot \exp(\mathrm{i}\,\boldsymbol{k}_{0-}\cdot\boldsymbol{r})\exp(-\mathrm{i}\,\boldsymbol{k}_{0+}\cdot\boldsymbol{r}')], \quad z \leq 0, z' < 0, \quad (14\mathrm{b})$$

with the reflection dyadics specified by

$$\underline{R}_e\{\varkappa_x,\varkappa_y\} = \varrho_{11}\boldsymbol{t}_-\boldsymbol{t}_+ + \varrho_{21}\boldsymbol{p}_{0-}\boldsymbol{t}_+ + \varrho_{12}\boldsymbol{t}_-\boldsymbol{p}_{0+} + \varrho_{22}\boldsymbol{p}_{0-}\boldsymbol{p}_{0+}, \quad (15\mathrm{a})$$

and

$$\underline{R}_m\{\varkappa_x,\varkappa_y\} = \varrho_{11}\boldsymbol{p}_{0-}\boldsymbol{t}_+ - \varrho_{21}\boldsymbol{t}_-\boldsymbol{t}_+ + \varrho_{12}\boldsymbol{p}_{0-}\boldsymbol{p}_{0+} - \varrho_{22}\boldsymbol{t}_-\boldsymbol{p}_{0+} \quad (15\mathrm{b})$$

In the transmission zone $z \geq d$, a similar strategy yields

$$\underline{G}_e(\boldsymbol{r},\boldsymbol{r}') = \underline{G}_{e\tau}(\boldsymbol{r},\boldsymbol{r}'),\ \underline{G}_m(\boldsymbol{r},\boldsymbol{r}') = \underline{G}_{m\tau}(\boldsymbol{r},\boldsymbol{r}'),\ z \geq d,\ z' < 0, \quad (16\mathrm{a,b})$$

where

$$\underline{G}_{e\tau}(\boldsymbol{r},\boldsymbol{r}') = (\mathrm{i}/8\pi^2)\int_{-\infty}^{\infty}\mathrm{d}\varkappa_x \int_{-\infty}^{\infty}\mathrm{d}\varkappa_y [(1/k_{0z})\underline{T}_e\{\varkappa_x,\varkappa_y\} \cdot \exp(-\mathrm{i}\,k_{0z}d)\exp(\mathrm{i}\,\boldsymbol{k}_{0+}\cdot\boldsymbol{r})\exp(-\mathrm{i}\,\boldsymbol{k}_{0+}\cdot\boldsymbol{r}')], \quad z \geq d,\ z' < 0, \quad (17\mathrm{a})$$

$$\underline{G}_{m\tau}(\boldsymbol{r},\boldsymbol{r}') = -(k_0/8\pi^2)\int_{-\infty}^{\infty}\mathrm{d}\varkappa_x \int_{-\infty}^{\infty}\mathrm{d}\varkappa_y [(1/k_{0z})\underline{T}_m\{\varkappa_x,\varkappa_y\} \cdot \exp(-\mathrm{i}\,k_{0z}d)\exp(\mathrm{i}\,\boldsymbol{k}_{0+}\cdot\boldsymbol{r})\exp(-\mathrm{i}\,\boldsymbol{k}_{0+}\cdot\boldsymbol{r}')], \quad z \geq d,\ z' < 0, \quad (17\mathrm{b})$$

with the transmission dyadics give as

$$\underline{T}_e\{\varkappa_x,\varkappa_y\} = \tau_{11}\boldsymbol{t}_+\boldsymbol{t}_+ + \tau_{21}\boldsymbol{p}_{0+}\boldsymbol{t}_+ + \tau_{12}\boldsymbol{t}_+\boldsymbol{p}_{0+} + \tau_{22}\boldsymbol{p}_{0+}\boldsymbol{p}_{0+}, \quad (18\mathrm{a})$$

and

$$\underline{T}_m\{\varkappa_x,\varkappa_y\} = \tau_{11}\boldsymbol{p}_{0+}\boldsymbol{t}_+ - \tau_{21}\boldsymbol{t}_+\boldsymbol{t}_+ + \tau_{12}\boldsymbol{p}_{0+}\boldsymbol{p}_{0+} - \tau_{22}\boldsymbol{t}_+\boldsymbol{p}_{0+}. \quad (18\mathrm{b})$$

Thus, the electric field $\boldsymbol{E}(\boldsymbol{r})$ of (8) can be determined in the reflection ($z \leq 0$) and the transmission ($z \geq d$) zones for any source current densities lying exclusively in the lower halfspace. The corresponding magnetic field $\boldsymbol{H}(\boldsymbol{r})$ can be determined using the Faraday equation in free space.

4. Asymptotic Evaluation of the Green's Dyadics

The Green's dyadics for the time-harmonic response of a chiral slab have been obtained in the spectral form. As can be readily surmised, the integrations on the right hand sides of (14a, b) and (17a, b) are well nigh impossible to perform analytically for general cases, and must be evaluated numerically. However, the asymptotic properties of these integrals are likely to be of interest within the context of electromagnetic shielding materials, and will be now explored.

Beginning with the reflection Green's dyadics (14a, b), one can notice that critical point (of the first kind) in the exponents on the right hand sides is [18]

$$\varkappa_{x,\mathrm{cr}} = (k_0/R_\varrho)(x-x'), \quad \varkappa_{y,\mathrm{cr}} = (k_0/R_\varrho)(y-y'), \quad k_{0z,\mathrm{cr}} = -(k_0/R_\varrho)(z+z'), \quad (19\mathrm{a–c})$$

where

$$R_\varrho = +\sqrt{(x-x')^2 + (y-y')^2 + (z+z')^2} \quad (19\mathrm{d})$$

There, in the limit $k_0 R \to \infty$, correct to order $(1/k_0 R)$, it can be seen that

$$\underline{G}_{e\varrho}(\boldsymbol{r},\boldsymbol{r}') \to \underline{R}_{e,\mathrm{cr}}\exp(\mathrm{i}\,k_0 R_\varrho)/4\pi R_\varrho, \quad z \leq 0,\ z' < 0, \quad (20\mathrm{a})$$

$$\underline{G}_{m\varrho}(\boldsymbol{r},\boldsymbol{r}') \to \mathrm{i}\,k_0\,\underline{R}_{m,\mathrm{cr}}\exp(\mathrm{i}\,k_0 R_\varrho)/4\pi R_\varrho, \quad z \leq 0,\ z' < 0, \quad (20\mathrm{b})$$

where

$$\underline{R}_{e,cr} = \underline{R}_e \{(k_0/R_\varrho)(x-x'), (k_0/R_\varrho)(y-y')\}, \quad (20\,c)$$

and

$$\underline{R}_{m,cr} = \underline{R}_m \{(k_0/R_\varrho)(x-x'), (k_0/R_\varrho)(y-y')\}. \quad (20\,d)$$

In the same fashion, the critical point for the asymptotic evaluation of the integrals in (17 a, b) is

$$\varkappa_{x,cr} = (k_0/R_\tau)(x-x'), \quad \varkappa_{y,cr} = (k_0/R_\tau)(y-y'),$$
$$k_{0z,cr} = (k_0/R_\tau)(z-z'-d), \quad (21\,a\!-\!c)$$

where

$$R_\tau = +\sqrt{(x-x')^2 + (y-y')^2 + (z-z'-d)^2}; \quad (21\,d)$$

hence, in the limit $k_0 R \to \infty$, correct to order $(1/k_0 R)$,

$$\underline{G}_{e\tau}(\boldsymbol{r},\boldsymbol{r}') \to \underline{T}_{e,cr} \exp(i k_0 R_\tau)/4\pi R_\tau,$$
$$z \ge d, \; z' < 0, \; z - z' \ge d, \quad (22\,a)$$

$$\underline{G}_{m\tau}(\boldsymbol{r},\boldsymbol{r}') \to i k_0 \underline{T}_{m,cr} \exp(i k_0 R_\tau)/4\pi R_\tau,$$
$$z \ge d, \; z' < 0, \; z - z' \ge d, \quad (22\,b)$$

in which

$$\underline{T}_{e,cr} = \underline{T}_e \{(k_0/R_\tau)(x-x'), (k_0/R_\tau)(y-y')\}, \quad (22\,c)$$

$$\underline{T}_{m,cr} = \underline{T}_m \{(k_0/R_\tau)(x-x'), (k_0/R_\tau)(y-y')\}. \quad (22\,d)$$

5. Example: Vertical Electric Dipole

As a simple example of the application of the previous section we consider a vertically directed point electric dipole, of magnitude p, located at $z' = h(<0)$; then, $\boldsymbol{J}(\boldsymbol{r}) = -i\omega p \boldsymbol{u}_z \delta(\boldsymbol{r} - h\boldsymbol{u}_z)$ and $\boldsymbol{K}(\boldsymbol{r}) = \boldsymbol{0}$.

Suppose the chiral slab did not exist; then for all points $z \ge d$ (actually for $z > h$), the electric field could be computed from (12 a) as

$$\boldsymbol{E}_{noslab}(\boldsymbol{r}) = (i/8\pi^2) p \omega^2 \mu_0 \int_{-\infty}^{\infty} d\varkappa_x \int_{-\infty}^{\infty} d\varkappa_y [(\varkappa/k_0 k_{0z}) \boldsymbol{p}_{0+} \cdot$$
$$\cdot \exp(i \boldsymbol{k}_{0+} \cdot \boldsymbol{r}) \exp(-i k_{0z} h)], \quad z \ge d, \; h < 0. \quad (23\,a)$$

But when the chiral slab is present, the electric field in the transmission zone is given by

$$\boldsymbol{E}_{slab}(\boldsymbol{r}) = (i/8\pi^2) p \omega^2 \mu_0$$
$$\cdot \int_{-\infty}^{\infty} d\varkappa_x \int_{-\infty}^{\infty} d\varkappa_y [(\varkappa/k_0 k_{0z})(\tau_{12} \boldsymbol{t}_+ + \tau_{22} \boldsymbol{p}_{0+}) \cdot$$
$$\cdot \exp(i \boldsymbol{k}_{0+} \cdot \boldsymbol{r}) \exp(-i k_{0z} h) \exp(-i k_{0z} d)],$$
$$z \ge d, \; h < 0. \quad (23\,b)$$

In the limit $k_0 r \to \infty$ with $z \gg d, z \gg |h|$ (23 a) can be approximated by

$$\boldsymbol{E}_{noslab}(r\boldsymbol{u}_r) \to -(\omega^2 \mu_0 p/4\pi r) \exp(i k_0 r) \cdot \quad (24\,a)$$
$$\cdot \exp(-i k_0 h \cos\theta) \boldsymbol{u}_r \times (\boldsymbol{u}_r \times \boldsymbol{u}_z), \; z \gg d, z \gg |h|,$$

while (23 b) can be shown to reduce to

$$\boldsymbol{E}_{slab}(r\boldsymbol{u}_r) \to -(\omega^2 \mu_0 p/4\pi r) \exp(i k_0 r)$$
$$\cdot \exp(-i k_0 h \cos\theta) \exp(-i k_0 d \cos\theta)$$
$$\cdot [\tau_{22}\{k_0^2 \sin^2\theta\} \boldsymbol{u}_r \times (\boldsymbol{u}_r \times \boldsymbol{u}_z) + \tau_{12}\{k_0^2 \sin^2\theta\} \boldsymbol{u}_r \times \boldsymbol{u}_z],$$
$$z \gg d, z \gg |h|, \quad (24\,b)$$

where $\boldsymbol{u}_r$ is the unit radial vector in the spherical coordinate system (r, θ, φ). From these expressions it is clear that the electric field in the far transmission zone ($k_0 r \to \infty, z \gg d, z \gg |h|$) is independent of the azimuth angle φ whether or not the chiral slab is present. More importantly, an observer in the far transmission zone, unaware of the chiral slab, would interpret (24 b) as the electric field due to an electric and a magnetic dipole, both vertically oriented and co-located at $\boldsymbol{r}' = h\boldsymbol{u}_z$; the appearance of the magnetic dipole moment must be credited solely to the handedness of the chiral slab. Furthermore, as is evident from (24 b), observers at different elevation angles θ will report different ratios of the magnitudes of the two (apparent) dipole moments. Next, whereas the $\boldsymbol{E}_{noslab}(r\boldsymbol{u}_r)$ of (24 a) is a linearly polarized plane wave in the far transmission zone, the $\boldsymbol{E}_{slab}(r\boldsymbol{u}_r)$ of (24 b) is an elliptically polarized plane wave in general. Finally, in a direction $\boldsymbol{u}_r$ in the far transmission zone, the effect of introducing the chiral slab is to reduce the transmitted power density per unit solid angle by a factor $[\tau_{22}^2\{k_0^2 \sin^2\theta\} + \tau_{12}^2\{k_0^2 \sin^2\theta\}]$.

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Book-Review · Buchbesprechung

Craig Scott, Modern Methods of Reflector Antenna Analysis and Design. Artech House, Boston/Mass. USA, 1990, 130 Seiten, 42 Bilder, 16 cm × 23 cm, gebunden, £ 47.–, ISBN 0-89006-419-9.

Dieser 15. Band der Antennenserie des Verlags eines mehr mathematisch orientierten Autors der US-Weltraumindustrie (Rockwell) ist mit der Absicht geschrieben (Vorwort), „eine verständliche Behandlung der neuen Reflektorantennen-Methoden (für Studenten) zu liefern und diese als Elemente so in die zeitgemäße Reflektorantennen-Theorie zu integrieren"... Um es vorwegzunehmen: Das Buch erfüllt zwar diesen (recht engen) Anspruch, aber die Erklärung der *Physik* der (Beugungs-)Vorgänge bleibt oft auf der Strecke. Wenn diese „theoretischen Elemente" in zehn Kapiteln gebracht werden, deren Anwendung sich überlappen, so erwartet man mehr an Beispielen, Vergleichen und Grafiken.

Das 1. Kapitel bringt Grundbegriffe („Reflektorantennen-Konzept"); ein Hinweis darauf, daß Vektoren durchgängig fett gedruckt sind, wäre sinnvoll gewesen. Das 2. Kapitel behandelt „Ludwigs Methode", d. h. die physikalische Optik (PO) ohne Randströme mit komplexen Feldansätzen, wobei das Erregerdiagramm per n-te Potenz des cos(θ) definiert wird; sie dient als erprobter Prüfstein für andere Methoden, aber sie trägt auch die Gefahr von Gitterkeulen in sich. Kapitel 3: Rusch's Methode, bezieht sogar lateral und axial aus dem Brennfeld verschobene Erreger ein, setzt aber Vorkenntnisse in Differentialgeometrie (kaum ingenieurmäßig!) voraus. Die „Jacobi-Bessel-Methode" (nach Galindo und Mittra) folgt in Kap. 4; hier werden zum Einbeziehen der Spiegelkrümmung (Strombelegungsmethode) die Zernike-Polynome (von 1934) herangezogen. Die Belegungsfunktion zur Fernfeldberechnung (illumination) besteht dabei aus einem Satz orthogonaler Funktionen und die Fernfeldlösung erfolgt über eine Rekursion des Strahlungsintegrals. Kap. 5 beschreibt die Fourier-Bessel-Theorie, die den Vorteil der mathematischen Vereinfachung trägt und auf der Anwendung der FFT fußt; sie wird am Beispiel der Brennfeldberechnung demonstriert. Die (sampling) Abtastmethode (nach Bucci '83) in Kap. 6 basiert auf einer modifizierten Fourier-Bessel-Variante, wobei die Belegung durch einer Fourier-Reihe genähert wird; hier wird besonders wenig auf die Physik eingegangen. Die „Methode der quadratischen Phase" (nach Pogorzelski '83) des Kap. 7s baut auf Ludwig's Methode auf, wobei der (schnell variierende) Phasenterm statt linear hier quadratisch angesetzt wird und eine partielle Integration zur Anwendung kommt. Die Lösung mittels Tchebyscheff-Polynom-Näherung erfolgt über eine Rekursion. Das 8. Kapitel über „Entwurf von Primärfokus-gespeiste Reflektoren" geht sogar auf defokussierte Systeme ein, deren Realisierung auf die Petzvalflächen führen. Zum Kap. 9: Entwurf von Doppelreflektor-Systemen: Wenn schon der dritte Satz eines solchen Kapitels auf die Literatur verweist, irritiert damit den Leser stark, so daß er sich fragen muß: Soll ich das dann überhaupt lesen? Hier werden die Einflüsse der drei und fünf geometrischen Parameter (zum Haupt- u. Subreflektor) auf den Entwurf von symmetrischen und unsymmetrischen („offset") Mehrspiegelsystemen diskutiert, wobei der Schwerpunkt auf der Randstrahlbedingung liegt. Daß die Bedingung zur Unterdrückung der Kreuzpolarisation „Mizugutch-Bedingung" heißt (seit 1976!) hat sich in manchen (US-)Kreisen noch nicht herumgesprochen (oder liegt da Absicht im Nichtzitieren?): Die Aperturfeld-Integration (AI) nach der GO füllt Kapitel 10, im Anhang A findet man die Ableitung zu den Oberflächen-Normalen und Flächenelementen und im Anhang B die Analyse des Subreflektor-Fernfeldes nach der GTD und der Entwicklung nach sphärischen Wellenfunktionen. Das Schlagwortverzeichnis ist sehr karg; eine Symbolliste fehlt.

Sicherlich eignet sich das Buch als Vorlesungs-Unterlage für TU-Studenten der höheren Semester, aber ob es in dieser Form (d. h. ohne Programmpakete o. ä.) auch „praktischen Ingenieuren" (siehe Vorwort) wirklich nützt, mag offenbleiben.

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