Metamaterials & Chirality

Akhlesh Lakhtakia

Department of Engineering Science & Mechanics
Pennsylvania State University

1600 hrs, February 28, 2007

Department of Physics, University of Pennsylvania

A. Lakhtakia

J.B.S. Haldane

The Creator, if he exists, has ...

PENNSTATE
A. Lakhtakia

J.B.S. Haldane

… an inordinate fondness for beetles.

A. Lakhtakia





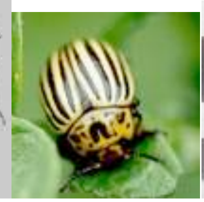




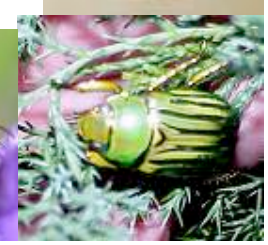


… an inordinate fondness for beetles.







PENNSTATE

A. Lakhtakia

Engineers

have had an inordinate fondness

for

composite materials…

A. Lakhtakia

… right from the Bronze Age.

PENNSTATE

1855

A. Lakhtakia

Composite Materials

Conspirator-in-Chief:
Tom G. Mackay

School of Mathematics, University of Edinburgh

A. Lakhtakia

Frontiers of Materials Research

A. Lakhtakia

Evolution of *Materials Research Frontiers*

- Material Properties (< 1980)

Evolution of *Materials Research Frontiers*

- Design for Functionality (ca.1980)

Evolution of *Materials Research Frontiers*

- Design for System Performance (ca. 2000)

A. Lakhtakia

Multifunctionality

Thanks: Chuck Bakis

PENNSTATE

A. Lakhtakia

Multifunctionality

<u>Performance Requirements on the Fuselage</u>

1. Light weight (for fuel efficiency)
2. High stiffness (resistance to deformation)
3. High strength (resistance to rupture)
4. High acoustic damping (quieter cabin)
5. Low thermal conductivity (less condensation; more humid cabin)

Multifunctionality

A. Lakhtakia

Performance Requirements on the Fuselage

1. Light weight (for fuel efficiency)
2. High stiffness (resistance to deformation)
3. High strength (resistance to rupture)
4. High acoustic damping (quieter cabin)
5. Low thermal conductivity (less condensation; more humid cabin)

Future: Conducting fibers for

(i) reinforcement
(ii) antennas
(iii) environmental sensing
(iv) structural health monitoring
(iv) morphing

Evolution of *Materials Research Frontiers*

- Material Properties (< 1980)
- Design for Functionality (ca.1980)
- Design for System Performance (ca. 2000)

PENNSTATE

1855

A. Lakhtakia

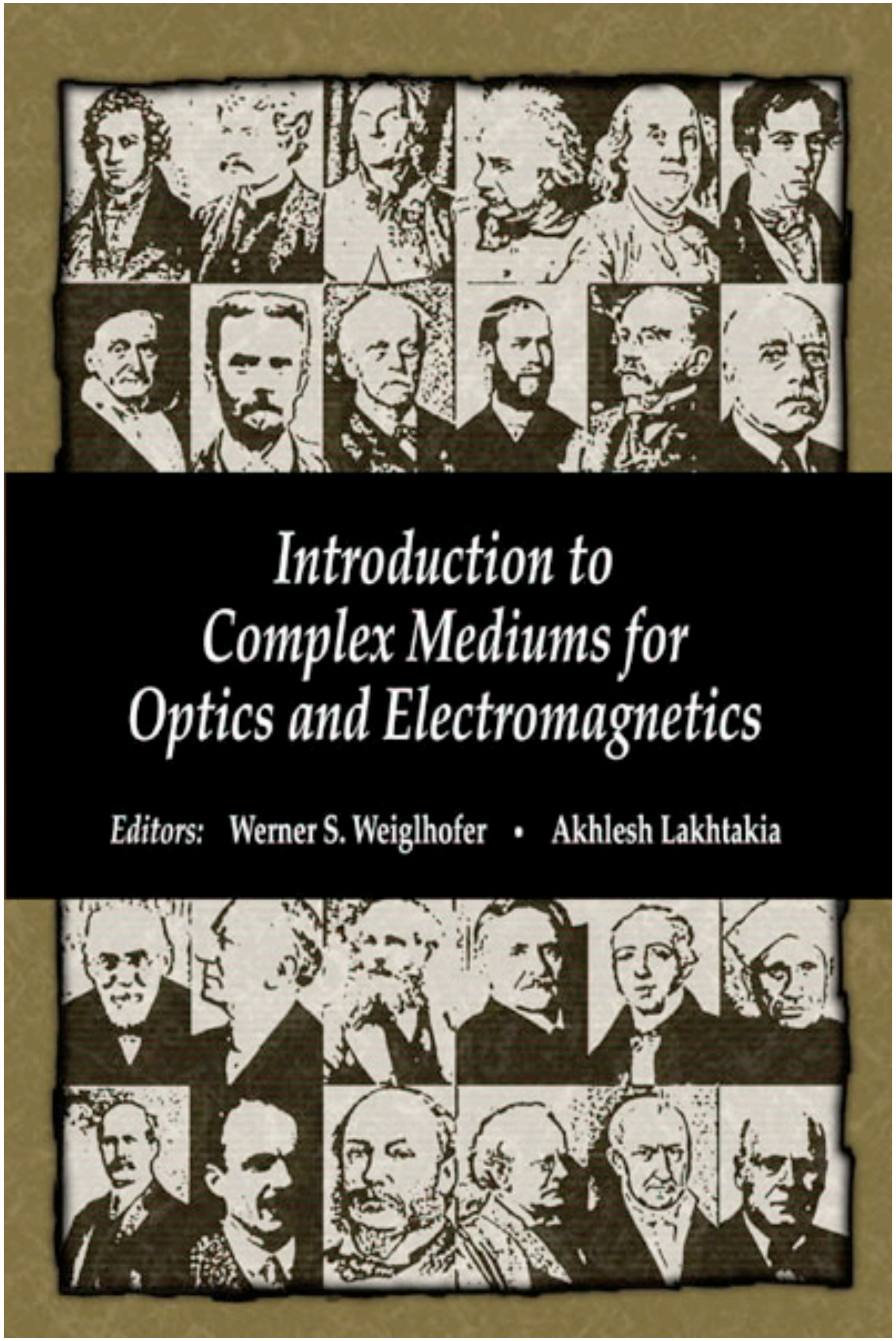
Metamaterials

A. Lakhtakia

Metamaterials

Rodger Walser

SPIE Press (2003)



Walser's Definition (2001/2)

- macroscopic composites having a manmade, three-dimensional, periodic cellular architecture designed to produce an optimized combination, not available in nature, of two or more responses to specific excitation

Walser's Definition (2001/2)

A. Lakhtakia

manmade

PENNSTATE

A. Lakhtakia

Walser’s Definition (2001/2)

three-dimensional

PENNSTATE

A. Lakhtakia

Walser's Definition (2001/2)

periodic

cellular

PENNSTATE

A. Lakhtakia

Walser's Definition (2001/2)

designed to produce an optimized combination of two or more responses to specific excitation

PENNSTATE

A. Lakhtakia

Walser's Definition (2001/2)

not

available in nature

Walser's Definition (2001/2)

not

available in nature

D.G. Stavenga, Invertebrate superposition eye-structures that behave like metamaterial with negative refractive index, *JEOS-RP* **1**, 06010 (2006).

Walser's Definition (2001/2)

- macroscopic composites having a manmade, three-dimensional, periodic cellular architecture designed to produce an optimized combination, not available in nature, of two or more responses to specific excitation

PENNSTATE

A. Lakhtakia

Working Definition

'Metamaterial'

– composite which exhibits properties:

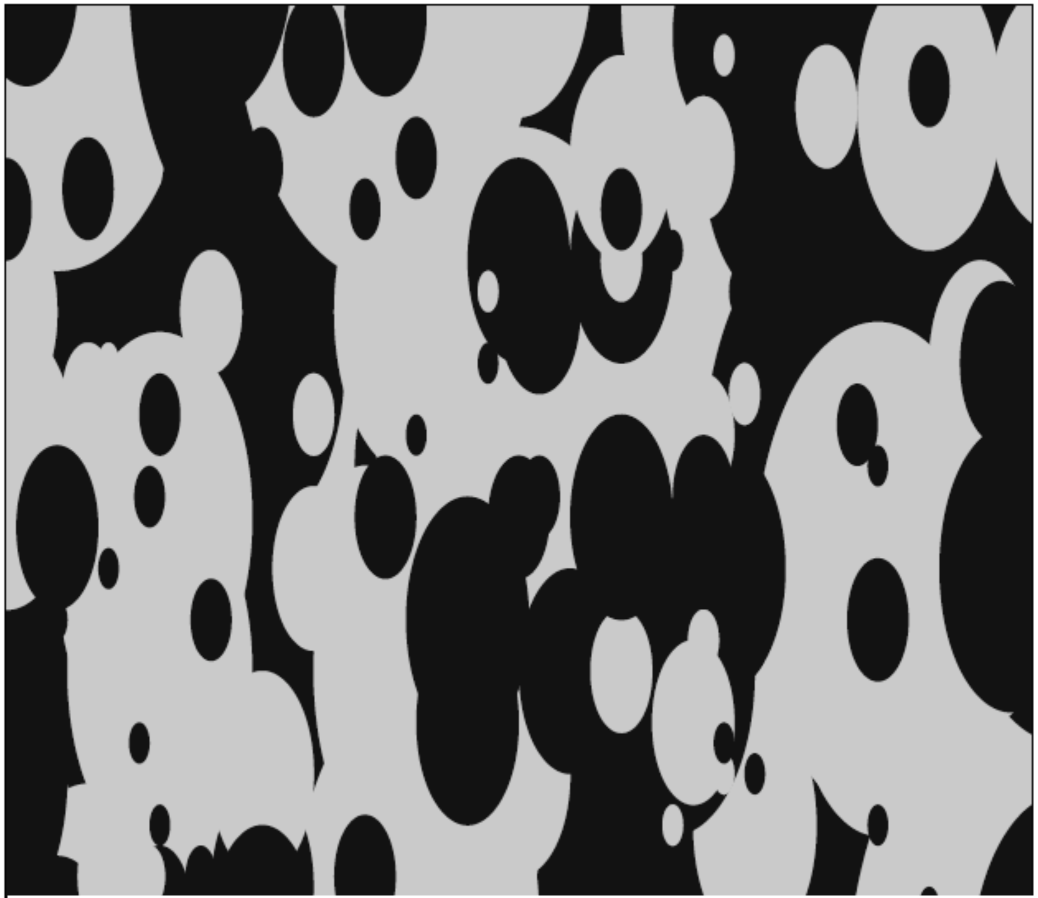
  * not observed in constituents

  or

  * enhanced relative to properties of constituents

A. Lakhtakia

Examples:
Particulate Composite Materials with ellipsoidal inclusions



$\lambda \gg$ inclusion size

Homogenizable Metamaterials

A. Lakhtakia

- Enhancement of group velocity

Homogenizable Metamaterials

- Enhancement of nonlinearity

Homogenizable Metamaterials

- Voigt wave propagation (degenerate eigenvectors)

Homogenizable Metamaterials

A. Lakhtakia

- Bianisotropy (e.g., Faraday chiral medium)

Homogenizable Metamaterials

- Negative phase velocity (isotropy/anisotropy)

Homogenizable Metamaterials

- Enhancement of group velocity
- Enhancement of nonlinearity
- Voigt wave propagation
- Bianisotropy
- Negative phase velocity

http://www.esm.psu.edu/~axl4/lakhtakia/documents/Mackay_06_6MRI.pdf

A. Lakhtakia

Composite Materials with Viscoelastic Stiffness Greater Than Diamond

T. Jaglinski,[1] D. Kochmann,[2] D. Stone,[3] R. S. Lakes[4]*

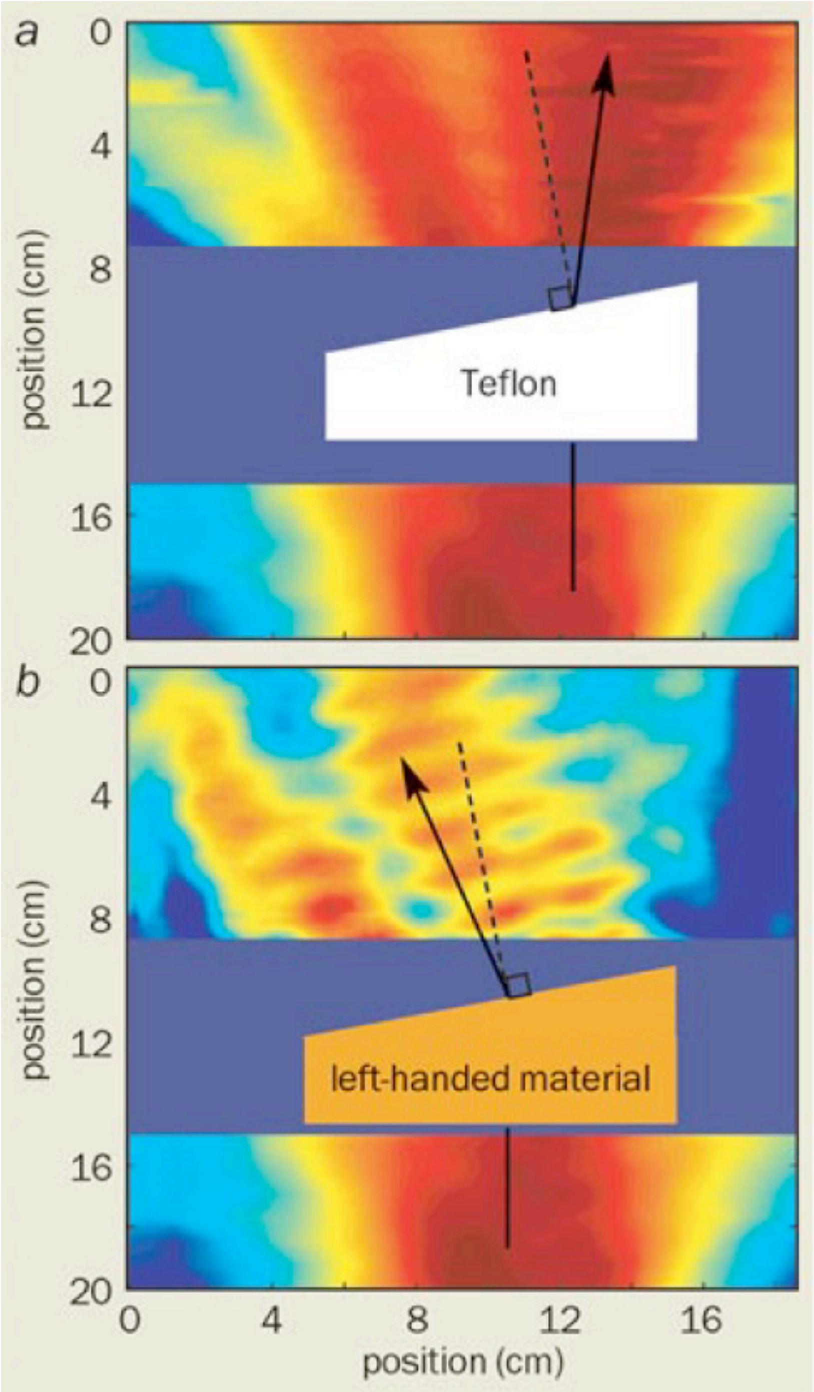
We show that composite materials can exhibit a viscoelastic modulus (Young's modulus) that is far greater than that of either constituent. The modulus, but not the strength, of the composite was observed to be substantially greater than that of diamond. These composites contain barium-titanate inclusions, which undergo a volume-change phase transformation if they are not constrained. In the composite, the inclusions are partially constrained by the surrounding metal matrix. The constraint stabilizes the negative bulk modulus (inverse compressibility) of the inclusions. This negative modulus arises from stored elastic energy in the inclusions, in contrast to periodic composite metamaterials that exhibit negative refraction by inertial resonant effects. Conventional composites with positive-stiffness constituents have aggregate properties bounded by a weighted average of constituent properties; their modulus cannot exceed that of the stiffest constituent.

2 FEBRUARY 2007 VOL 315 SCIENCE

Negative Phase Velocity

PENNSTATE

A. Lakhtakia



material with n > 0

material with n < 0

Adapted from David Smith's website

What to make of it?

INSTITUTE OF PHYSICS PUBLISHING EUROPEAN JOURNAL OF PHYSICS

Eur. J. Phys. **23** (2002) 353–359 PII: S0143-0807(02)31789-6

The negative index of refraction demystified

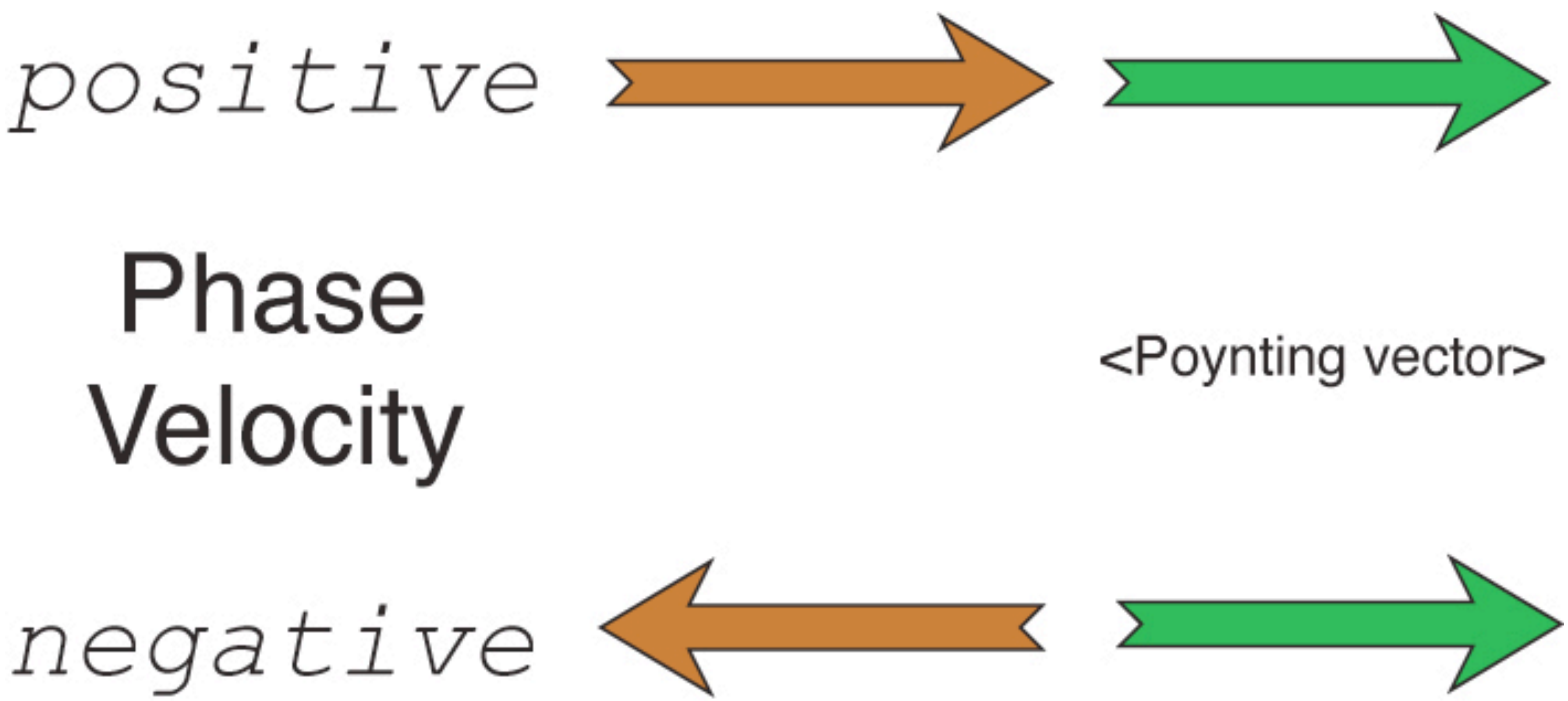
Martin W McCall[1,4], Akhlesh Lakhtakia[2] and Werner S Weiglhofer[3]

Two Important Quantities

- Phase velocity vector
- Time-averaged Poynting vector
  = direction of energy flow & attenuation

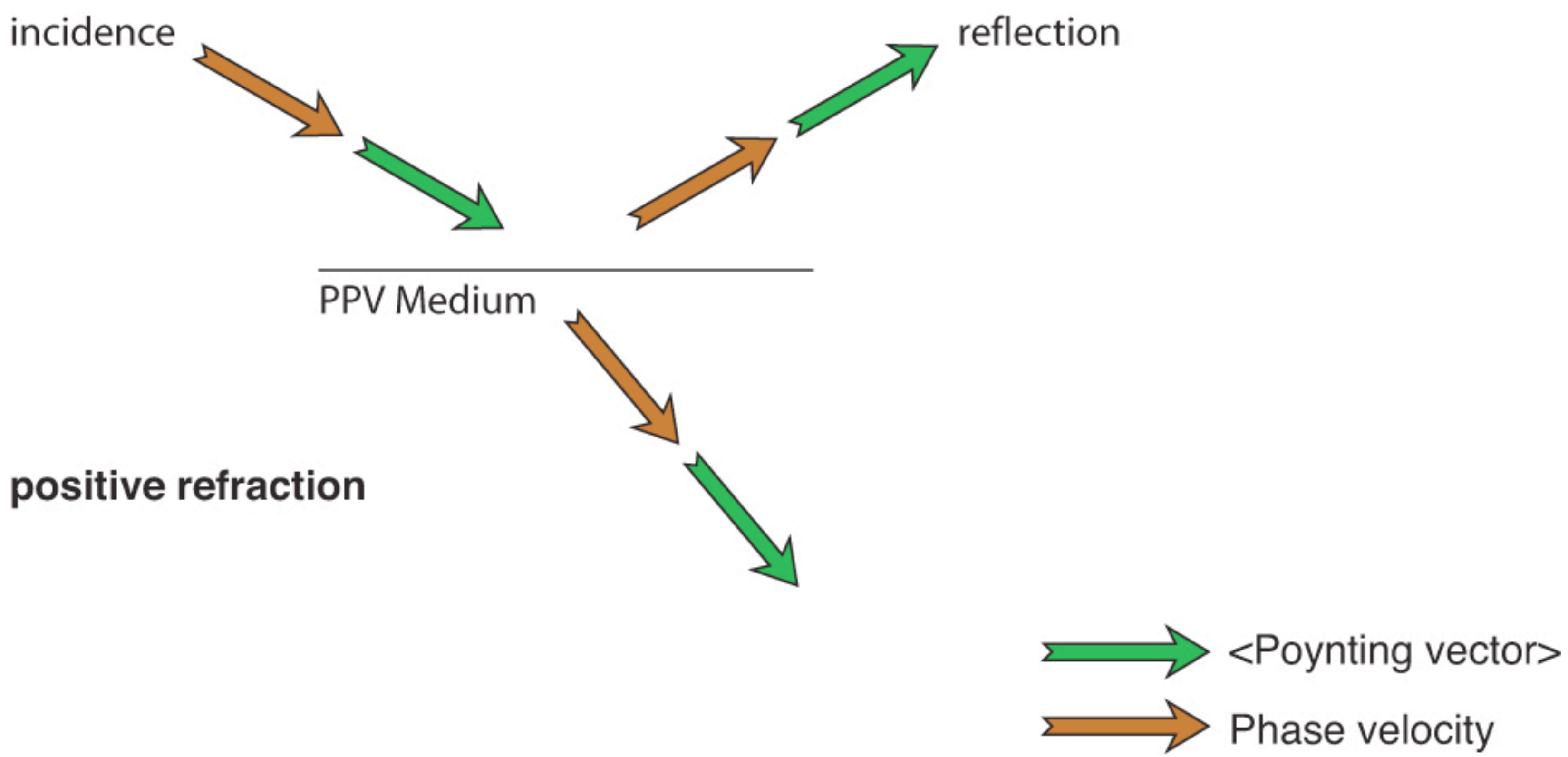
A. Lakhtakia

Positive/Negative *Phase Velocity Medium*



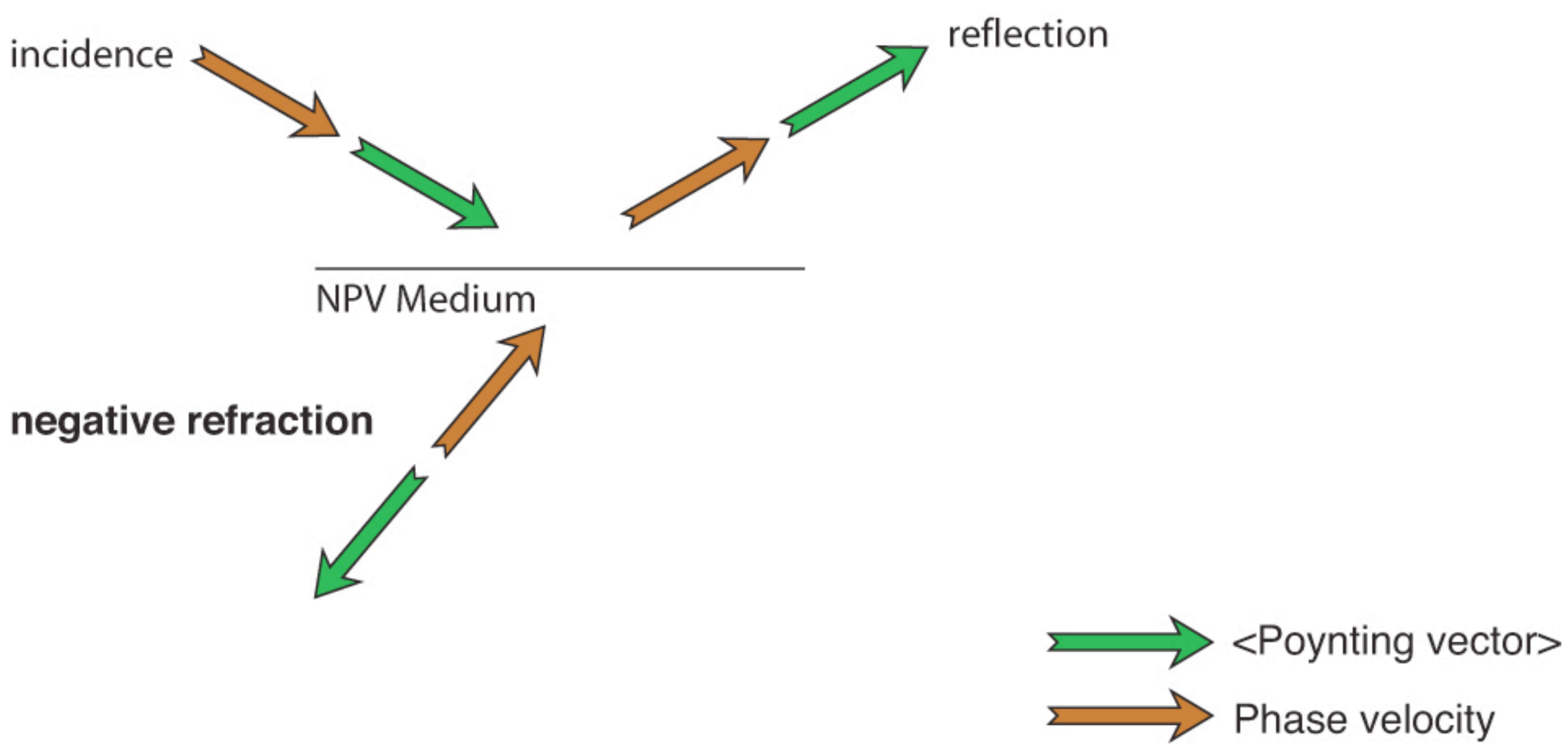
A. Lakhtakia

Positive Refraction by Isotropic Medium

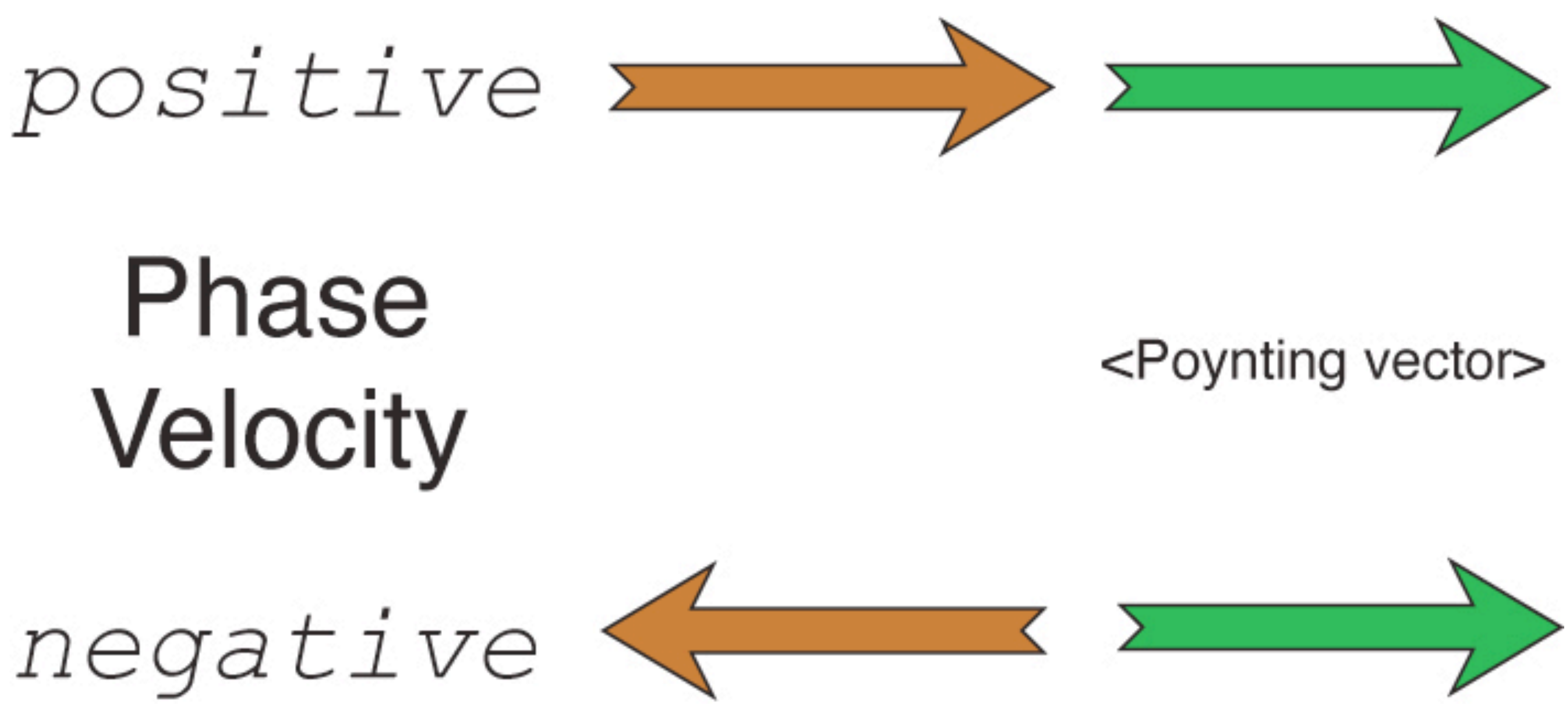


A. Lakhtakia

Negative Refraction by Isotropic Medium



NPV in Simple Mediums



NPV in Bianisotropic Mediums

PHYSICAL REVIEW E **69**, 026602 (2004)

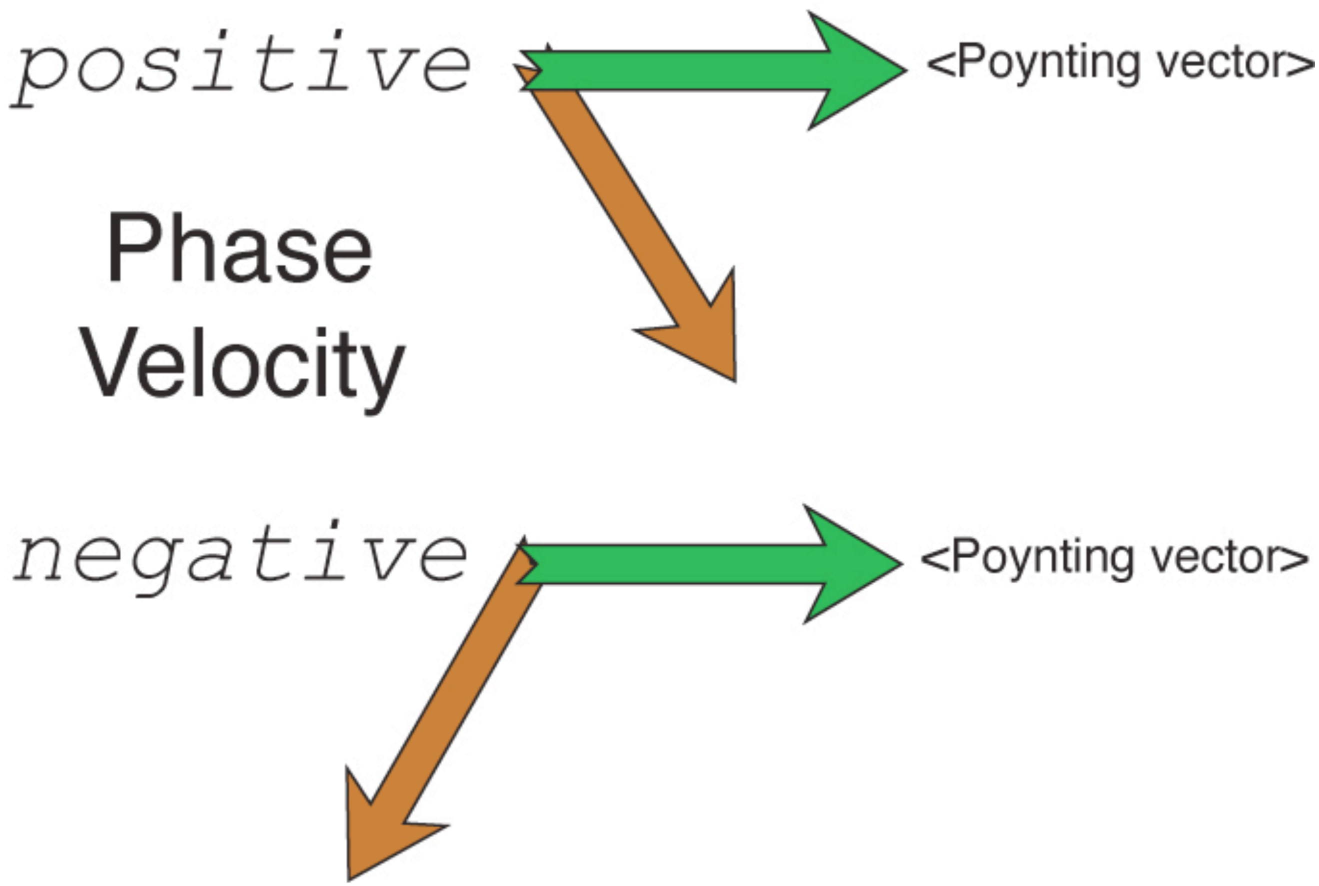
Plane waves with negative phase velocity in Faraday chiral mediums

Tom G. Mackay*
School of Mathematics, James Clerk Maxwell Building, The King's Buildings, University of Edinburgh, Edinburgh EH9 3JZ, United Kingdom

Akhlesh Lakhtakia†
CATMAS — Computational and Theoretical Materials Sciences Group, Department of Engineering Science and Mechanics, Pennsylvania State University, University Park, Pennsylvania 16802-6812, USA
(Received 18 July 2003; published 10 February 2004)

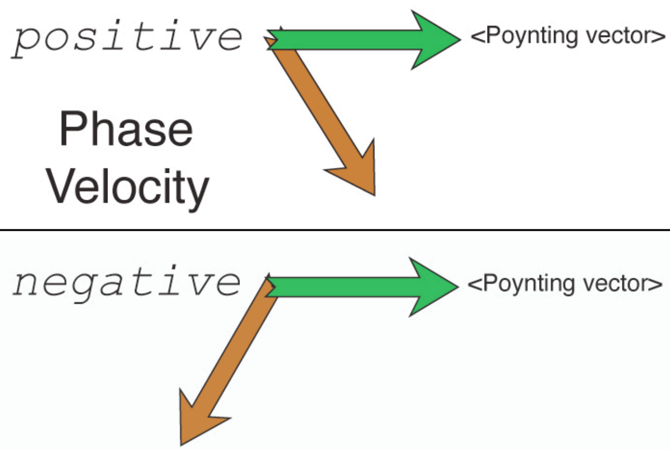
NPV in Bianisotropic Mediums

A. Lakhtakia



A. Lakhtakia

NPV in Bianisotropic Mediums



$$\mathrm{Re}\{\mathbf{k}\}\bullet<\mathbf{P}> \;\; > 0$$

$$\mathrm{Re}\{\mathbf{k}\}\bullet<\mathbf{P}> \;\; < 0$$

k = wave vector

A. Lakhtakia

Chirality

A. Lakhtakia

Types of Chirality

A. Lakhtakia

Types of Chirality

(a) Microscopic/Microstructural

(i) Isotropic

$$\underline{D}(\underline{r}) = \epsilon_0\, \epsilon\, \underline{E}(\underline{r}) + i\sqrt{\epsilon_0 \mu_0}\, \xi\, \underline{H}(\underline{r})$$

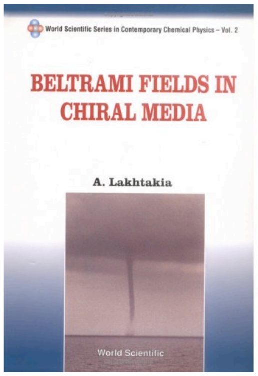
$$\underline{B}(\underline{r}) = -i\sqrt{\epsilon_0 \mu_0}\, \xi\, \underline{E}(\underline{r}) + \mu_0\, \mu\, \underline{H}(\underline{r})$$

Frequency-domain constitutive equations

A. Lakhtakia

Types of Chirality

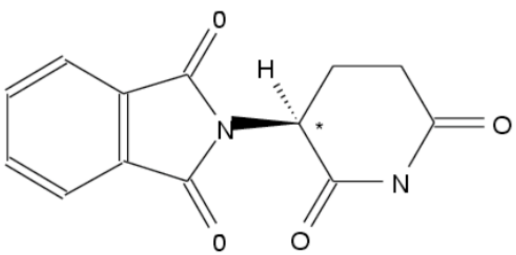
(a) Microscopic/Microstructural



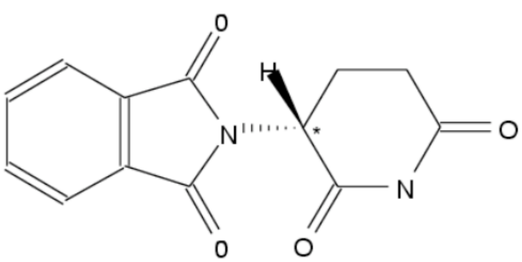
(i) Isotropic

$$\underline{D}(\underline{r}) = \epsilon_0\, \epsilon\, \underline{E}(\underline{r}) + i\sqrt{\epsilon_0 \mu_0}\, \xi\, \underline{H}(\underline{r})$$

$$\underline{B}(\underline{r}) = -i\sqrt{\epsilon_0 \mu_0}\, \xi\, \underline{E}(\underline{r}) + \mu_0\, \mu\, \underline{H}(\underline{r})$$

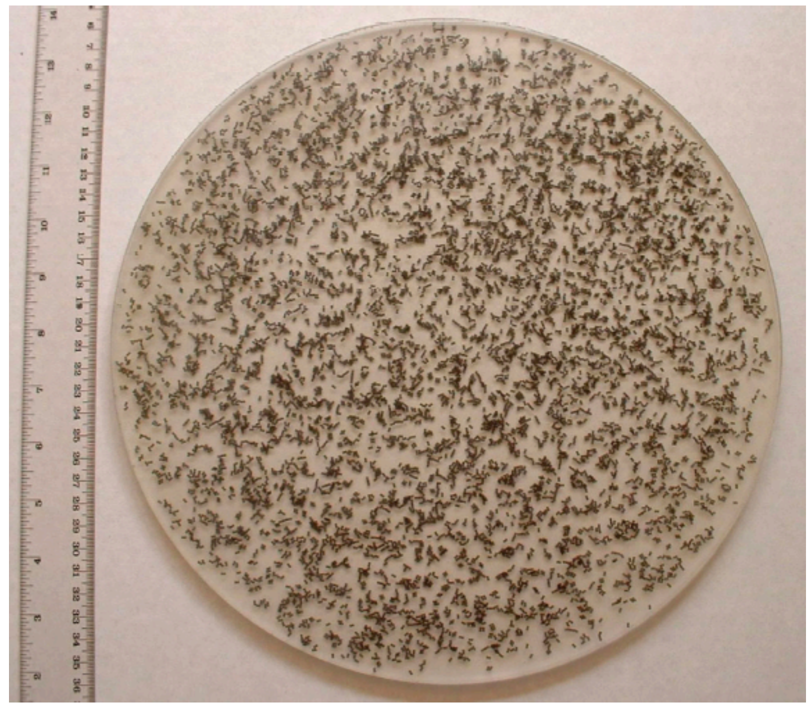


Enantiomère (S) : tératogène



Enantiomère (R) : Non–toxique

Courtesy:
Á. Gómez,
Univ. Cantabria



PENNSTATE

A. Lakhtakia

Types of Chirality

(a) Microscopic/Microstructural

(i) Isotropic

$$\underline{D}(\underline{r}) = \epsilon_0\, \epsilon\, \underline{E}(\underline{r}) + i\sqrt{\epsilon_0\mu_0}\, \xi\, \underline{H}(\underline{r})$$
$$\underline{B}(\underline{r}) = -i\sqrt{\epsilon_0\mu_0}\, \xi\, \underline{E}(\underline{r}) + \mu_0\, \mu\, \underline{H}(\underline{r})$$

(ii) Faraday chiral

$$\underline{D}(\underline{r}) = \underline{\underline{\epsilon}} \bullet \underline{E}(\underline{r}) + \underline{\underline{\xi}} \bullet \underline{H}(\underline{r})$$
$$\underline{B}(\underline{r}) = -\underline{\underline{\xi}} \bullet \underline{E}(\underline{r}) + \underline{\underline{\mu}} \bullet \underline{H}(\underline{r})$$

$$\underline{\underline{\epsilon}} = \epsilon_0 \left[\epsilon\, \underline{\underline{I}} - i\epsilon_g\, \hat{\underline{z}} \times \underline{\underline{I}} + (\epsilon_z - \epsilon)\, \hat{\underline{z}}\, \hat{\underline{z}} \right]$$
$$\underline{\underline{\xi}} = i\sqrt{\epsilon_0\mu_0} \left[\xi\, \underline{\underline{I}} - i\xi_g\, \hat{\underline{z}} \times \underline{\underline{I}} + (\xi_z - \xi)\, \hat{\underline{z}}\, \hat{\underline{z}} \right]$$
$$\underline{\underline{\mu}} = \mu_0 \left[\mu\, \underline{\underline{I}} - i\mu_g\, \hat{\underline{z}} \times \underline{\underline{I}} + (\mu_z - \mu)\, \hat{\underline{z}}\, \hat{\underline{z}} \right]$$

Frequency-domain constitutive equations

Types of Chirality

(a) Microscopic/Microstructural

(i) Isotropic

$$\underline{D}(\underline{r}) = \epsilon_0\,\epsilon\,\underline{E}(\underline{r}) + i\sqrt{\epsilon_0\mu_0}\,\xi\,\underline{H}(\underline{r})$$
$$\underline{B}(\underline{r}) = -i\sqrt{\epsilon_0\mu_0}\,\xi\,\underline{E}(\underline{r}) + \mu_0\,\mu\,\underline{H}(\underline{r})$$

(ii) Faraday chiral

$$\underline{D}(\underline{r}) = \underline{\underline{\epsilon}}\bullet\underline{E}(\underline{r}) + \underline{\underline{\xi}}\bullet\underline{H}(\underline{r})$$
$$\underline{B}(\underline{r}) = -\underline{\underline{\xi}}\bullet\underline{E}(\underline{r}) + \underline{\underline{\mu}}\bullet\underline{H}(\underline{r})$$

$$\underline{\underline{\epsilon}} = \epsilon_0\left[\epsilon\,\underline{\underline{I}} - i\epsilon_g\,\hat{\underline{z}}\times\underline{\underline{I}} + (\epsilon_z-\epsilon)\,\hat{\underline{z}}\,\hat{\underline{z}}\right]$$
$$\underline{\underline{\xi}} = i\sqrt{\epsilon_0\mu_0}\left[\xi\,\underline{\underline{I}} - i\xi_g\,\hat{\underline{z}}\times\underline{\underline{I}} + (\xi_z-\xi)\,\hat{\underline{z}}\,\hat{\underline{z}}\right]$$
$$\underline{\underline{\mu}} = \mu_0\left[\mu\,\underline{\underline{I}} - i\mu_g\,\hat{\underline{z}}\times\underline{\underline{I}} + (\mu_z-\mu)\,\hat{\underline{z}}\,\hat{\underline{z}}\right]$$

(iii) Nonhomogeneous Variants

Frequency-domain constitutive equations

A. Lakhtakia

Types of Chirality

(b) Macrostructural

$$\underline{D}(\underline{r}) = \underline{\underline{S}}(z) \bullet \left[\underline{\underline{\epsilon}}_{ref} \bullet \underline{\underline{S}}^T(z) \bullet \underline{E}(\underline{r}) + \underline{\underline{\xi}}_{ref} \bullet \underline{\underline{S}}^T(z) \bullet \underline{H}(\underline{r}) \right]$$

$$\underline{B}(\underline{r}) = \underline{\underline{S}}(z) \bullet \left[\underline{\underline{\zeta}}_{ref} \bullet \underline{\underline{S}}^T(z) \bullet \underline{E}(\underline{r}) + \underline{\underline{\mu}}_{ref} \bullet \underline{\underline{S}}^T(z) \bullet \underline{H}(\underline{r}) \right]$$

Linear Bianisotropic Materials

$$\begin{aligned}
\underline{\underline{S}}_x(z) &= \mathbf{u}_x\mathbf{u}_x + (\mathbf{u}_y\mathbf{u}_y + \mathbf{u}_z\mathbf{u}_z)\cos\xi(z) \\
&\quad + (\mathbf{u}_z\mathbf{u}_y - \mathbf{u}_y\mathbf{u}_z)\sin\xi(z)\,, \\
\underline{\underline{S}}_y(z) &= \mathbf{u}_y\mathbf{u}_y + (\mathbf{u}_x\mathbf{u}_x + \mathbf{u}_z\mathbf{u}_z)\cos\tau(z) \\
&\quad + (\mathbf{u}_z\mathbf{u}_x - \mathbf{u}_x\mathbf{u}_z)\sin\tau(z)\,, \\
\underline{\underline{S}}_z(z) &= \mathbf{u}_z\mathbf{u}_z + (\mathbf{u}_x\mathbf{u}_x + \mathbf{u}_y\mathbf{u}_y)\cos\zeta(z) \\
&\quad + (\mathbf{u}_y\mathbf{u}_x - \mathbf{u}_x\mathbf{u}_y)\sin\zeta(z)\,.
\end{aligned}$$

A. Lakhtakia

Types of Chirality

(b) Macrostructural

Dielectric Materials

$$\mathbf{D}(\mathbf{r},\omega) = \epsilon_0 \, \underline{\underline{\epsilon}}_r(z,\omega) \bullet \mathbf{E}(\mathbf{r},\omega)$$

$$= \epsilon_0 \, \underline{\underline{S}}(z) \bullet \underline{\underline{\epsilon}}_{ref}(\omega) \bullet \underline{\underline{S}}^T(z) \bullet \mathbf{E}(\mathbf{r},\omega) \, ,$$

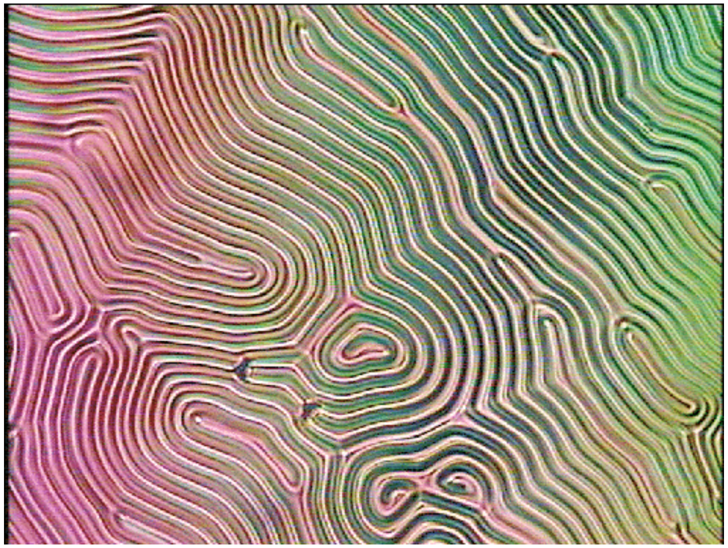
$$\mathbf{B}(\mathbf{r},\omega) = \mu_0 \, \mathbf{H}(\mathbf{r},\omega) \, .$$

Local Orthorhombicity

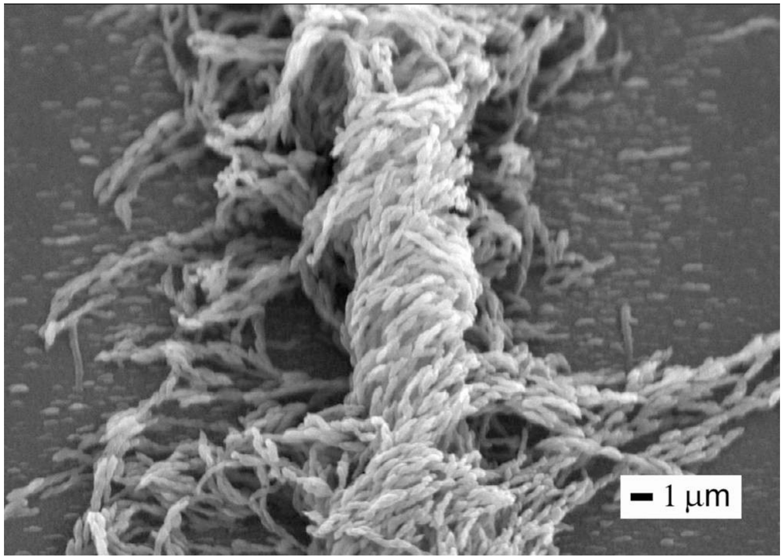
A. Lakhtakia

Types of Chirality

(b) Macrostructural



http://www.mc2.chalmers.se/pl/lc/engelska/gallery/fingerprint.html



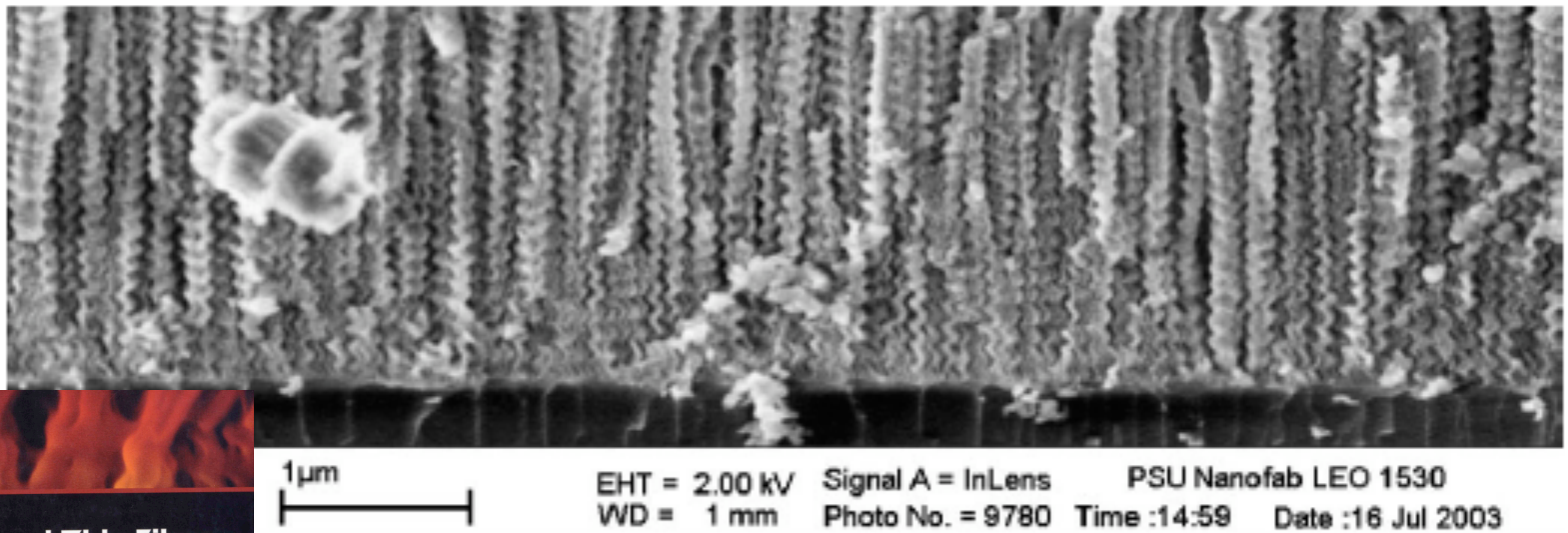
http://www.lcd.kent.edu/images/4.htm

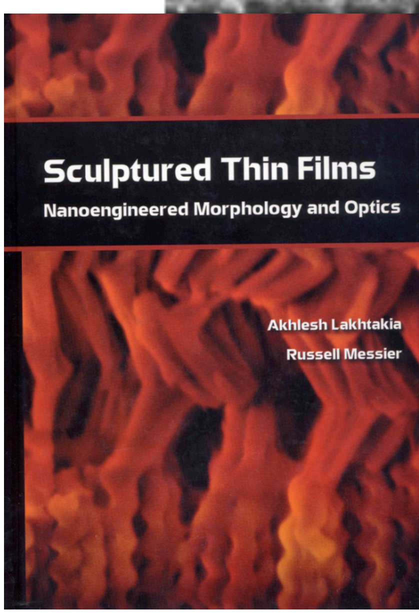
Cholesteric LC with helical axis in the substrate plane

Twisted-grain polymer morphology due to a choleesteric LC host

Types of Chirality

(b) Macrostructural





Chiral sculptured thin film

Isotropic Chiral Medium

PENNSTATE

A. Lakhtakia

Isotropic Chiral Medium

$$\underline{D}(\underline{r}) = \epsilon_0\, \epsilon\, \underline{E}(\underline{r}) + i\sqrt{\epsilon_0\mu_0}\, \xi\, \underline{H}(\underline{r})$$

$$\underline{B}(\underline{r}) = -i\sqrt{\epsilon_0\mu_0}\, \xi\, \underline{E}(\underline{r}) + \mu_0\, \mu\, \underline{H}(\underline{r})$$

PLANE WAVES WITH NEGATIVE PHASE VELOCITY IN ISOTROPIC CHIRAL MEDIUMS

Tom G. Mackay
School of Mathematics
University of Edinburgh
James Clerk Maxwell Building
King's Buildings
Edinburgh EH9 3JZ, UK

120 MICROWAVE AND OPTICAL TECHNOLOGY LETTERS / Vol. 45, No. 2, April 20 2005

Isotropic Chiral Medium

Plane waves

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_0 \exp(ik\,\hat{\mathbf{k}} \bullet \mathbf{r})$$

$$\mathbf{H}(\mathbf{r}) = \mathbf{H}_0 \exp(ik\,\hat{\mathbf{k}} \bullet \mathbf{r})$$

NPV condition

$$\mathrm{Re}\,[\mathbf{k}] \bullet \langle \mathbf{P} \rangle < 0$$

4 wavenumbers

$$k^{(i)} = -\omega\sqrt{\epsilon_0\mu_0}\left(\sqrt{\epsilon\mu} + \xi\right)$$

$$k^{(ii)} = \omega\sqrt{\epsilon_0\mu_0}\left(\sqrt{\epsilon\mu} - \xi\right)$$

$$k^{(iii)} = -\omega\sqrt{\epsilon_0\mu_0}\left(\sqrt{\epsilon\mu} - \xi\right)$$

$$k^{(iv)} = \omega\sqrt{\epsilon_0\mu_0}\left(\sqrt{\epsilon\mu} + \xi\right)$$

Isotropic Chiral Medium

2 NPV Conditions

$$\mathrm{Re}\left\{\sqrt{\epsilon\mu}+\xi\right\}\times\mathrm{Re}\left\{\sqrt{\frac{\epsilon^*}{\mu^*}}\right\}<0 \qquad \text{for} \qquad k=k^{(i),(iv)}$$

$$\mathrm{Re}\left\{\sqrt{\epsilon\mu}-\xi\right\}\times\mathrm{Re}\left\{\sqrt{\frac{\epsilon^*}{\mu^*}}\right\}<0 \qquad \text{for} \qquad k=k^{(ii),(iii)}$$

Negligible dissipation

$$\sqrt{\epsilon\mu}<-\xi \qquad \text{for} \qquad k=k^{(i),(iv)}$$

$$\sqrt{\epsilon\mu}<\xi \qquad \text{for} \qquad k=k^{(ii),(iii)}$$

Isotropic Chiral Medium

A. Lakhtakia

Planewave propagation in a specific direction

2 Beltrami modes

Both PPV	$\mathrm{Re}[k] > 0$
Both NPV	$\mathrm{Re}[k] < 0$
Both IPV	$\mathrm{Re}[k] = 0$
1 NPV, 1 PPV	
1 IPV, 1 PPV	
1 IPV, 1NPV	

A. Lakhtakia

Isotropic Chiral Medium

Advantage: Birefringence

Refraction can create two channels.

Isotropic Chiral Medium

Advantage: Birefringence

Refraction can create two channels.

Challenge: Can ξ be large enough

so that

one channel is NPV,

the other PPV?

PENNSTATE

1855

A. Lakhtakia

Faraday Chiral Medium

A. Lakhtakia

Faraday Chiral Medium

PHYSICAL REVIEW E **69**, 026602 (2004)

Plane waves with negative phase velocity in Faraday chiral mediums

Tom G. Mackay*
School of Mathematics, James Clerk Maxwell Building, The King's Buildings, University of Edinburgh, Edinburgh EH9 3JZ, United Kingdom

Akhlesh Lakhtakia†
CATMAS — Computational and Theoretical Materials Sciences Group, Department of Engineering Science and Mechanics, Pennsylvania State University, University Park, Pennsylvania 16802-6812, USA
(Received 18 July 2003; published 10 February 2004)

A. Lakhtakia

Faraday Chiral Medium

$$\underline{D}(\underline{r}) = \underline{\underline{\epsilon}} \bullet \underline{E}(\underline{r}) + \underline{\underline{\xi}} \bullet \underline{H}(\underline{r})$$
$$\underline{B}(\underline{r}) = -\underline{\underline{\xi}} \bullet \underline{E}(\underline{r}) + \underline{\underline{\mu}} \bullet \underline{H}(\underline{r})$$

Mixture of

ICM and magnetically biased ferrite

$$\underline{\underline{\epsilon}} = \epsilon_0 \left[\epsilon \, \underline{\underline{I}} - i\epsilon_g \, \hat{\underline{z}} \times \underline{\underline{I}} + (\epsilon_z - \epsilon) \, \hat{\underline{z}} \, \hat{\underline{z}} \right]$$
$$\underline{\underline{\xi}} = i\sqrt{\epsilon_0 \mu_0} \left[\xi \, \underline{\underline{I}} - i\xi_g \, \hat{\underline{z}} \times \underline{\underline{I}} + (\xi_z - \xi) \, \hat{\underline{z}} \, \hat{\underline{z}} \right]$$
$$\underline{\underline{\mu}} = \mu_0 \left[\mu \, \underline{\underline{I}} - i\mu_g \, \hat{\underline{z}} \times \underline{\underline{I}} + (\mu_z - \mu) \, \hat{\underline{z}} \, \hat{\underline{z}} \right]$$

Faraday Chiral Medium

A. Lakhtakia

Plane waves

$$\underline{E}(\underline{r}) = \underline{E}_0 \, \exp(i k_o \tilde{k} \, \hat{\underline{u}} \bullet \underline{r})$$

$$\underline{H}(\underline{r}) = \underline{H}_0 \, \exp(i k_o \tilde{k} \, \hat{\underline{u}} \bullet \underline{r})$$

Relative wavenumber

$$\tilde{k} = \tilde{k}_R + i \tilde{k}_I \,, \qquad (\tilde{k}_R, \tilde{k}_I \in \mathbb{R})$$

Direction of propagation

$$\hat{\underline{u}} = \hat{\underline{x}} \sin\theta + \hat{\underline{z}} \cos\theta$$

$$\langle \underline{P}(\underline{r}) \rangle = \frac{1}{2} \exp\left(-2 k_o \tilde{k}_I \, \hat{\underline{u}} \bullet \underline{r}\right) \mathrm{Re}\left\{ \underline{E}_0 \times \left[(\underline{\underline{\mu}}^{-1})^* \bullet \left(\sqrt{\epsilon_o \mu_o} \tilde{k}^* \, \hat{\underline{u}} \times \underline{E}_0^* + \underline{\underline{\xi}}^* \bullet \underline{E}_0^* \right) \right] \right\}$$

$$w = 2 \eta_o \exp\left(2 k_o \tilde{k}_I \, \hat{\underline{u}} \bullet \underline{r} \right) |E_{0y}|^{-2} \, \tilde{k}_R \, \hat{\underline{u}} \bullet \langle \underline{P}(\underline{r}) \rangle$$

NPV condition

$$\tilde{k}_R \, \hat{\underline{u}} \bullet \langle \underline{P}(\underline{r}) \rangle < 0 \Rightarrow w < 0$$

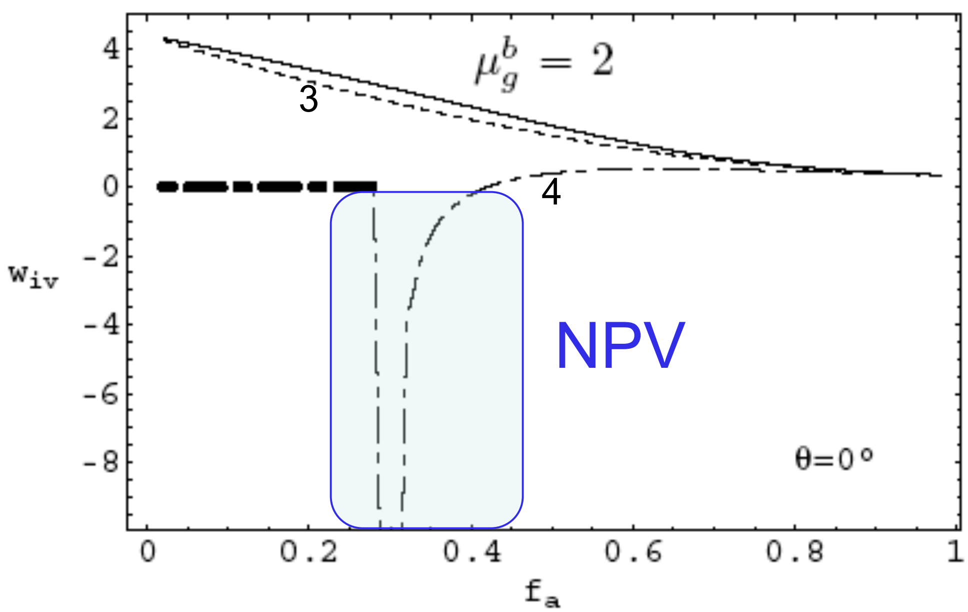
Faraday Chiral Medium

A. Lakhtakia

ICM **Ferrite**

$\epsilon^a = 3.2,\ \xi^a = 2.4,\ \mu^a = 2;$ $\epsilon^b = 2.2,\ \mu^b = 3.5,\ \mu_z^b = 1,\ \mu_g^b \in [0, 4].$

4 wavenumbers



PENNSTATE

1855

A. Lakhtakia

Macrostructurally Chiral Medium

PENNSTATE
A. Lakhtakia

Ferrosmectic Slab

$$\left.\begin{array}{l}\underline{\underline{\epsilon}}(\mathbf{r}) = \epsilon_0 \, \underline{\underline{S}}_z \bullet \underline{\underline{S}}_y \bullet \left[\epsilon_a \, \mathbf{u}_z\mathbf{u}_z + \epsilon_b \, \mathbf{u}_x\mathbf{u}_x \, + \epsilon_c \, \mathbf{u}_y\mathbf{u}_y\right] \bullet \underline{\underline{S}}_y^T \bullet \underline{\underline{S}}_z^T \\ \underline{\underline{\mu}}(\mathbf{r}) = \mu_0 \, \underline{\underline{S}}_z \bullet \underline{\underline{S}}_y \bullet \left[\mu_a \, \mathbf{u}_z\mathbf{u}_z + \mu_b \, \mathbf{u}_x\mathbf{u}_x \, + \mu_c \, \mathbf{u}_y\mathbf{u}_y\right] \bullet \underline{\underline{S}}_y^T \bullet \underline{\underline{S}}_z^T \end{array}\right\} , \quad 0 \le z \le L$$

Tilt Dyadic

$$\underline{\underline{S}}_y = \mathbf{u}_y\mathbf{u}_y + (\mathbf{u}_x\mathbf{u}_x + \mathbf{u}_z\mathbf{u}_z) \cos\chi + (\mathbf{u}_z\mathbf{u}_x - \mathbf{u}_x\mathbf{u}_z) \sin\chi$$

Rotation (Helicity) Dyadic

$$\underline{\underline{S}}_z = \mathbf{u}_z\mathbf{u}_z + (\mathbf{u}_x\mathbf{u}_x + \mathbf{u}_y\mathbf{u}_y) \cos(h\pi z/\Omega) + (\mathbf{u}_y\mathbf{u}_x - \mathbf{u}_x\mathbf{u}_y) \sin(h\pi z/\Omega)$$

h = +1 or -1

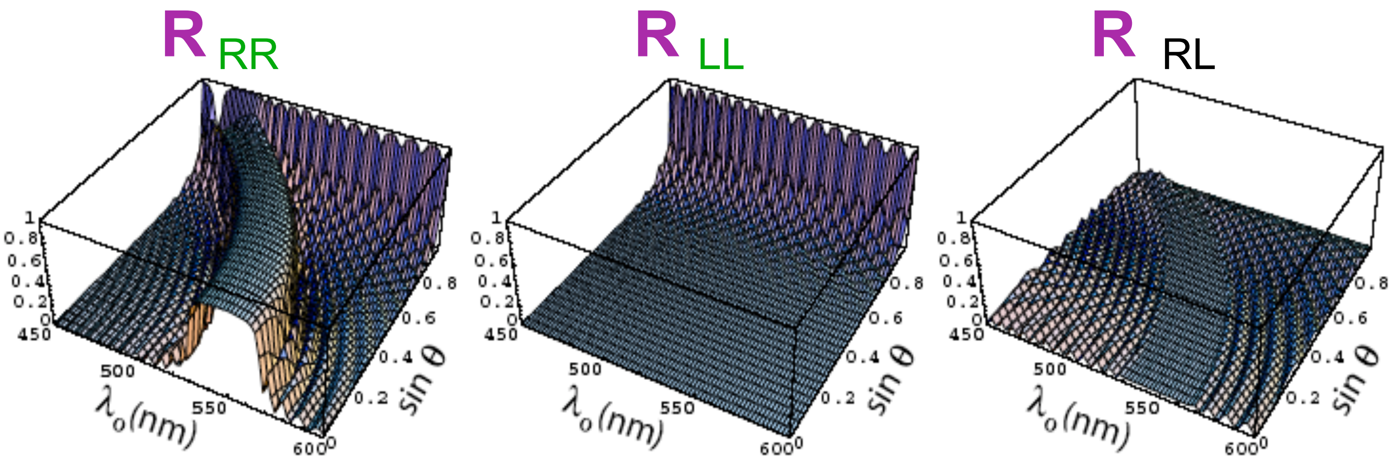
$$\mathbf{E}(\mathbf{r}) = \tilde{\mathbf{E}}(z) \exp\left[i\kappa(x\cos\psi + y\sin\psi)\right]$$

$$\mathbf{H}(\mathbf{r}) = \tilde{\mathbf{H}}(z) \exp\left[i\kappa(x\cos\psi + y\sin\psi)\right]$$

Ferrosmectic Slab

A. Lakhtakia

h = +1 $\quad Re[\varepsilon_{a,b,c}] > 0, \quad Re[\mu_{a,b,c}] > 0$



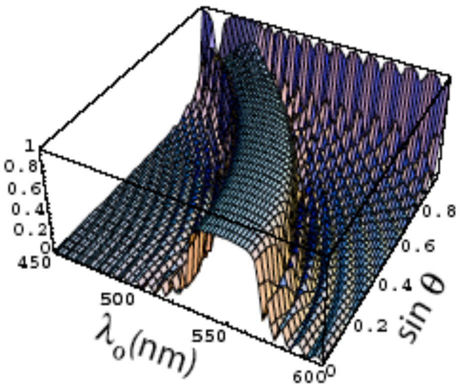
PENNSTATE

A. Lakhtakia

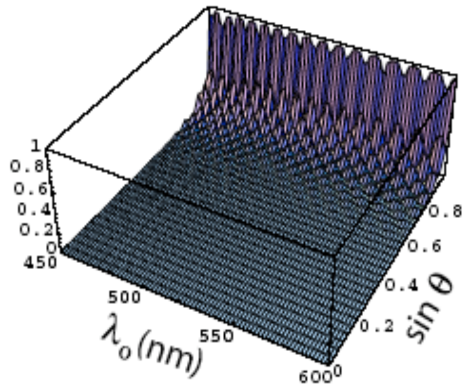
Ferrosmectic Slab

h = +1 Re[$\varepsilon_{a,b,c}$] > 0, Re[$\mu_{a,b,c}$] > 0

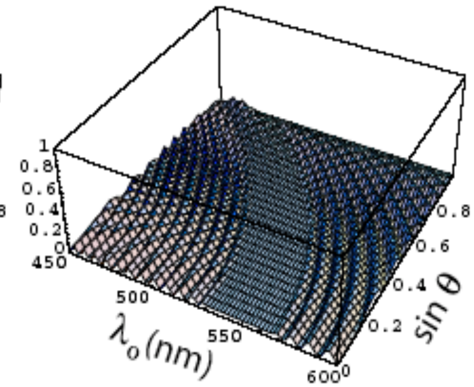
R_{RR}



R_{LL}

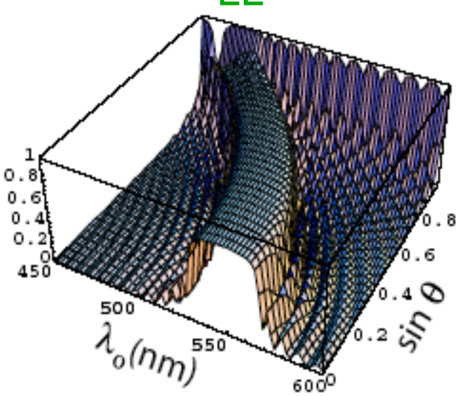


R_{RL}

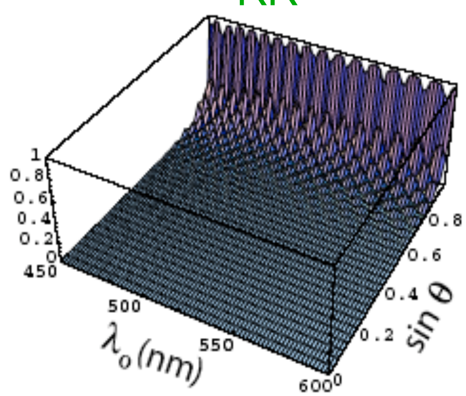


h = -1 Re[$\varepsilon_{a,b,c}$] > 0, Re[$\mu_{a,b,c}$] > 0

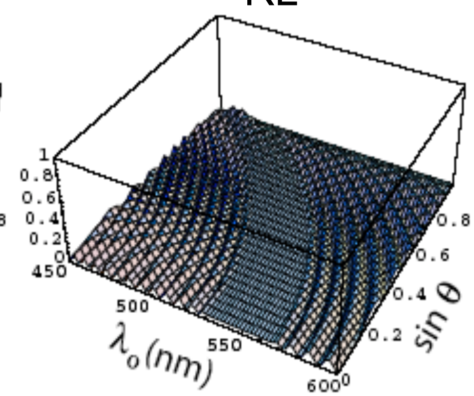
R_{LL}



R_{RR}



R_{RL}

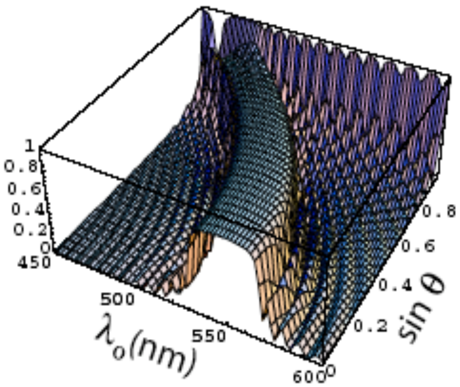


PENNSTATE
A. Lakhtakia

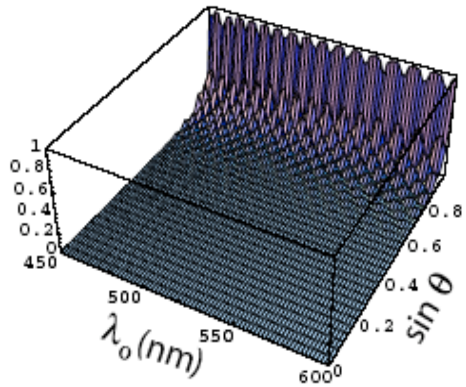
Ferrosmectic Slab

$h = +1$ $\mathrm{Re}[\varepsilon_{a,b,c}] > 0, \ \mathrm{Re}[\mu_{a,b,c}] > 0$

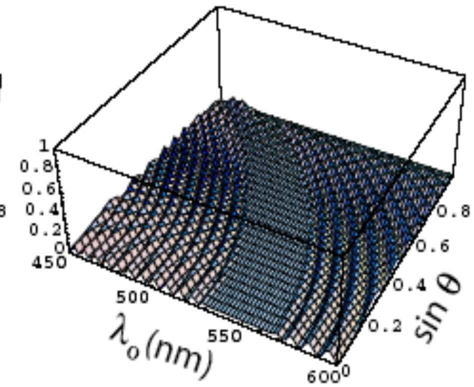
R_{RR}



R_{LL}



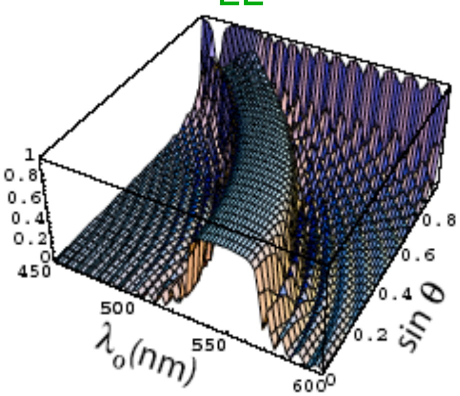
R_{RL}



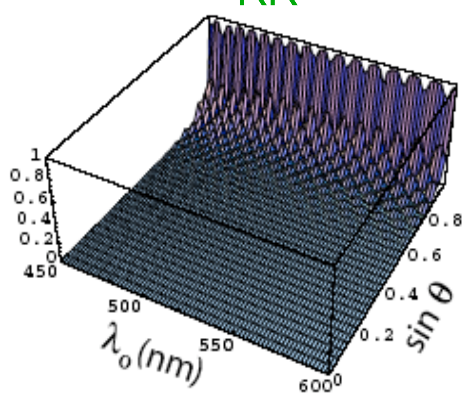
$h = +1$ $\mathrm{Re}[\varepsilon_{a,b,c}] < 0, \ \mathrm{Re}[\mu_{a,b,c}] < 0$

+ rotate slab by 180 deg

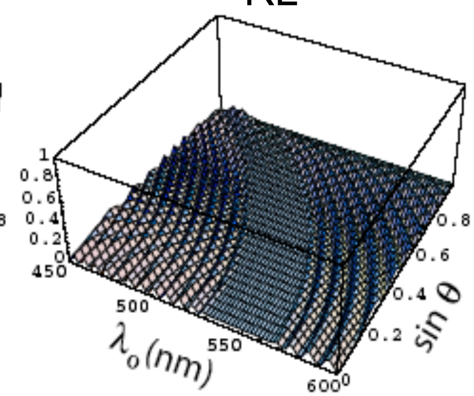
R_{LL}



R_{RR}



R_{RL}



Ferrosmectic Slab

A. Lakhtakia

Sign Reversal of Re[ε] and Re[μ]

+ Rotation of slab by 180 deg

Handedness Reversal of Chiral Structure

ADVANCED MATERIALS

Reversal of Circular Bragg Phenomenon in Ferrocholesteric Materials with Negative Real Permittivities and Permeabilities

By *Akhlesh Lakhtakia**

Adv. Mater. **2002**, *14*, No. 6, March 18 447

A. Lakhtakia

Ferrosmectic Slab

Sign Reversal of Re[$\underline{\varepsilon}$] and Re[$\underline{\mu}$]

+ Rotation of slab by 180 deg

Handedness Reversal of Chiral Structure

But that's not the entire truth!

Ferrosmectic Slab

A. Lakhtakia

Sign Reversal of Re[ε] and Re[μ]

+ Rotation of slab by 180 deg

Handedness Reversal of Chiral Structure

Handedness reversal of circular Bragg phenomenon due to negative real permittivity and permeability

Akhlesh Lakhtakia

CATMAS, Department of Engineering Science and Mechanics, Pennsylvania State University, University Park, PA 16802-6812

Received January 17, 2003; Revised February 24, 2003

7 April 2003 / Vol. 11, No. 7 / OPTICS EXPRESS 716

But that's not the entire truth!

Ferrosmectic Slab

A. Lakhtakia

$$\{\ h \leftrightarrow -h\,,\, \psi \leftrightarrow -\psi\ \} \Rightarrow \left\{ \begin{array}{ll} r_{LL} \leftrightarrow r_{RR}\,, & r_{LR} \leftrightarrow r_{RL} \\ t_{LL} \leftrightarrow t_{RR}\,, & t_{LR} \leftrightarrow t_{RL} \end{array} \right\}$$

$$\left\{ \begin{array}{c} \mathrm{Re}\left[\epsilon_{a,b,c}\right] \rightarrow -\mathrm{Re}\left[\epsilon_{a,b,c}\right] \\ \mathrm{Re}\left[\mu_{a,b,c}\right] \rightarrow -\mathrm{Re}\left[\mu_{a,b,c}\right] \\ \psi \rightarrow \pi + \psi \end{array} \right\}$$

$$\Rightarrow \left\{ \begin{array}{llll} r_{LL} \rightarrow r^*_{RR}, & r_{RR} \rightarrow r^*_{LL}, & r_{LR} \rightarrow r^*_{RL}, & r_{RL} \rightarrow r^*_{LR} \\ t_{LL} \rightarrow t^*_{RR}, & t_{RR} \rightarrow t^*_{LL}, & t_{LR} \rightarrow t^*_{RL}, & t_{RL} \rightarrow t^*_{LR} \end{array} \right\}$$

Phase reversal - additional

Conjugation Symmetry

PENNSTATE

A. Lakhtakia

Conjugation Symmetry

CONJUGATION SYMMETRY IN LINEAR ELECTROMAGNETISM IN EXTENSION OF MATERIALS WITH NEGATIVE REAL PERMITTIVITY AND PERMEABILITY SCALARS

Akhlesh Lakhtakia
CATMAS—Computational & Theoretical Materials Sciences Group
Dept. of Engineering Science & Mechanics
212 Earth & Engineering Sciences Building
Pennsylvania State University, University Park, PA 16802-6812

160 MICROWAVE AND OPTICAL TECHNOLOGY LETTERS / Vol. 40, No. 2, January 20 2004

Conjugation Symmetry

$$\underline{D}(\underline{r},\omega) = \underline{\underline{\hat{\epsilon}}}(\underline{r},\omega) \bullet \underline{E}(\underline{r},\omega) + \underline{\underline{\xi}}(\underline{r},\omega) \bullet \underline{H}(\underline{r},\omega)$$

$$\underline{B}(\underline{r},\omega) = \underline{\underline{\zeta}}(\underline{r},\omega) \bullet \underline{E}(\underline{r},\omega) + \underline{\underline{\mu}}(\underline{r},\omega) \bullet \underline{H}(\underline{r},\omega)$$

$$\left\{\underline{\underline{\hat{\epsilon}}} \to -\underline{\underline{\hat{\epsilon}}}^*,\ \underline{\underline{\mu}} \to -\underline{\underline{\mu}}^*,\ \underline{\underline{\xi}} \to -\underline{\underline{\xi}}^*,\ \zeta \to -\underline{\underline{\zeta}}^*\right\}$$

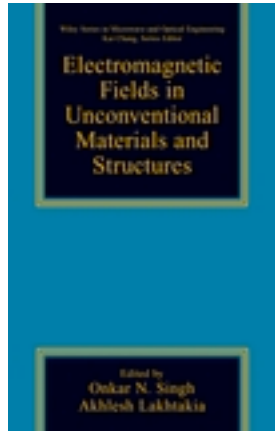
→

$$\{\underline{E} \to \underline{E}^*,\ \underline{H} \to \underline{H}^*,\ \underline{D} \to -\underline{D}^*,\ \underline{B} \to -\underline{B}^*\}$$

Assessment

A. Lakhtakia

Assessment



- Anisotropy: the direction-dependent contraction of space and absorption
- Chirality: the twisting of space
- Nonhomogeneity: the redirection of energy into different directions by material interfaces
- Nonlinearity: the emission of absorbed energy at (generally) some other frequency

A. Lakhtakia

Assessment

Geometry (structure) is integral to complex materials.

PENNSTATE

A. Lakhtakia

Assessment

Geometry (structure) is integral to complex materials.

Geometry begets chirality and anisotropy.

PENNSTATE

1855

A. Lakhtakia

Assessment

Geometry (structure) is integral to complex materials.

Geometry begets chirality and anisotropy.

Question: How important are chirality and anisotropy?

Assessment

Answer: **Isotropic Chiral Materials**
Faraday Chiral Materials

Polarization adjustment
like retro-rockets

Macrostructural Chiral Materials

Polarization filtering

A. Lakhtakia

Assessment

“Large” anisotropy is “easy” to design for and achieve.

“Large” isotropic chirality is not “easy” to design for and achieve.

PENNSTATE

A. Lakhtakia

Curl Enhancer

Assessment

A. Lakhtakia

"Large" isotropic chirality is not "easy" to design for and achieve.

Focus on nihility

$$\varepsilon = 0, \mu = 0$$

International Journal of Infrared and Millimeter Waves, Vol. 23, No. 6, June 2002 (© 2002)

AN ELECTROMAGNETIC TRINITY FROM "NEGATIVE PERMITTIVITY" AND "NEGATIVE PERMEABILITY"*

Akhlesh Lakhtakia


"That's all Folks!"