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THE ESSENCE OF THE BREWSTER PHENOMENON: BREWSTER WAVENUMBERS

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It is common to teach the Brewster angle as a zero-reflection angle, this fashion, beginning from the 1950s, repeated in an article published in this journal earlier this year.[1] Teaching the Brewster angle as a zero-reflection angle is like telling only an incomplete introduction to a story. Brewster himself would not have recognized this angle to be a zero-reflection angle, as I pointed out some years ago.[2] Instead, his purely experimental work convinced Brewster to think of his angle as a polarizing angle. Great ideas always have more in them than meets the eye. The polarizing angle comes about because of a specific decoupling of the reflected plane waves from the incident plane waves, and the essence of the Brewster phenomenon are the Brewster wavenumbers.

Suppose a plane wave of arbitrary polarization state were launched in free space. Suppose next that it were incident on a planar interface with an arbitrary homogeneous linear medium. Let the interface be the plane $z = 0$. Let the incident wave vector lie in the xz plane and make an angle θ_{inc} with the z axis. The incident plane wave can be thought of as comprising two components: one whose electric field phasor is polarized along the y axis and has an amplitude A_1, and the other whose electric field phasor is in the xz plane and has an amplitude A_2. The reflected plane wave also has similar components with electric field phasor amplitudes R_1 and R_2. One can then write

$$\begin{bmatrix} R_1(\theta_{inc}) \\ R_2(\theta_{inc}) \end{bmatrix} = \begin{bmatrix} r_{11}(\theta_{inc}) & r_{12}(\theta_{inc}) \\ r_{21}(\theta_{inc}) & r_{22}(\theta_{inc}) \end{bmatrix} \begin{bmatrix} A_1(\theta_{inc}) \\ A_2(\theta_{inc}) \end{bmatrix} \tag{1}$$

Suppose next that the matrix on the right side is singular for a particular angle of incidence; i.e., there exists an angle θ_B such that

$$r_{11}(\theta_B)\, r_{22}(\theta_B) = r_{12}(\theta_B)\, r_{21}(\theta_B). \tag{2}$$

In that eventuality, the ratio $R_1(\theta_B)/R_2(\theta_B)$ shall turn out to be independent of the ratio $A_1(\theta_B)/A_2(\theta_B)$.

θ_B is the Brewster angle. When it is the angle of incidence, the polarization state of the reflected plane wave is fixed, and it does not depend on the polarization state of the incident plane wave. The Brewster angle is thus, as Brewster astutely noted, the polarizing angle. Enterprising teachers can relate equation (2) to matrices as well.

Multiply $\cos\theta_B$ by the free space wavenumber and you get the Brewster wavenumber. The concept of Brewster wavenumber can be extended to the case when free space above is replaced by a linear homogeneous medium.[3] The Brewster wavenumber has acquired technological merit, as has been amply demonstrated in elastodynamics.[4,5]

Teaching undergraduate students that the Brewster angle is a zero-reflection angle is not proper. It is historically incorrect and it is intellectually impoverished. True, we cannot expect most undergraduate students to understand the richness of the Brewster phenomenon. Yet, after introducing the zero-reflection angle, the sage on the stage should inform the students that more gems lie beyond that operational definition.

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