

# A UNIDIRECTIONALLY CONDUCTING SCREEN IS A SUPER-BREWSTER SCREEN

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## KEY TERMS

Unidirectionally conducting screen, Brewster screen, bianisotropic media

## ABSTRACT

Expressions for the plane-wave response of a unidirectionally conducting screen (UCS) embedded in a linear, homogeneous, bianisotropic substance are obtained. The ratio of the complex amplitudes of the two reflected plane waves is shown to be always independent of the ratio of the complex amplitudes of the two incident plane waves; likewise, for the transmitted plane waves. Thus, a UCS is a super-Brewster screen. © 1993 John Wiley & Sons, Inc.

## 1. INTRODUCTION

The objective of this communication is to prove analytically that a unidirectionally conducting screen (UCS) is a *super-Brewster* screen. Thus, this communication has two distinct points of origin—Torald di Francia's UCS [1] and Brewster's polarizing angle [2]—and its specific aim is to provide a connection between the two.

Around the end of the year 1812, Sir David Brewster became successful in a series of experiments that greatly assisted in the elucidation of the nature of polarized light. In particular, he deduced that when unpolarized light is incident on an optically smooth, planar air-dielectric interface at a certain angle with respect to normal to the interface, the reflected light is completely polarized [2]. This particular angle came to be known as the *Brewster angle*, and the phenomenon has been widely utilized in the construction of *Brewster polarizers* [3].

The concept of the Brewster angle has widened in the last few years into that of the *Brewster conditions*. A general scheme for Brewster conditions relevant to the planar interface of two homogeneous, nondiffusive, linear bianisotropic media has been recently given by Lakhtakia [4], who has also cataloged several earlier investigations for specific bimaterial interfaces.

In 1956, Torald di Francia [1] introduced the UCS as an infinitesimally thin screen that is perfectly conducting in one tangential direction and perfectly insulating in the other. Scattering properties of a UCS in free space were thereafter investigated by, among others, Karp and Karal [5], Seshadri [6, 7] and Monzon [8, 9].

The plane-wave scattering response of a UCS immersed in a homogeneous biisotropic medium was recently reported in the pages of this journal [10]. In that work, the electromagnetic field was decomposed into a set of six Snel-compatible plane waves. Two of these six are supposed to be incident on the screen, two are reflected off the screen, while the remaining two constitute the transmitted field. The ratio of the amplitudes of the two *reflected* plane waves was derived as being independent of the ratio of the amplitudes of the two *incident* plane waves; thus, the UCS was shown to act as a *Brewster screen*. The ratio of the amplitudes of the two *transmitted* plane waves also turned out to be independent of

the ratio of the amplitudes of the two *incident* plane waves. Because of these properties, it can be stated that an UCS embedded in any linear, homogeneous, *biisotropic* electromagnetic matter is a *super-Brewster* screen.

The aim of this communication is to extend the result of [10]: It will be proved here that an UCS embedded in any linear, homogeneous, Lorentz-covariant, *bianisotropic* electromagnetic matter [11] is a super-Brewster screen. This will be done using results obtainable from the  $4 \times 4$  matrix method commonly used in optics and electromagnetics [4, 12, 13].

## 2. HOMOGENEOUS LORENTZ-COVARIANT CONTINUA

Let all space be occupied by a spatially local, nondiffusive, homogeneous, linear, bianisotropic medium. By requiring that all field equations be generally covariant, Post [11] obtained constitutive relations for these materials. In the frequency domain [ $e^{-i\omega t}$ ], these relations can be specified as

$$\mathbf{D}(x, y, z) = \mathbf{e}_p \cdot \mathbf{E}(x, y, z) + \boldsymbol{\alpha}_p \cdot \mathbf{B}(x, y, z), \quad (1a)$$

$$\mathbf{H}(x, y, z) = \mathbf{b}_p \cdot \mathbf{E}(x, y, z) + \mathbf{m}_p^{-1} \cdot \mathbf{B}(x, y, z), \quad (1b)$$

where  $\mathbf{e}_p$ , etc., are three-dimensional dyadics, while  $\mathbf{m}_p^{-1}$  is the inverse of  $\mathbf{m}_p$ . Relations (1a) and (1b) are not very convenient for the satisfaction of boundary value problems, because boundary conditions are specified in terms of the tangential components of the  $\mathbf{E}$  and the  $\mathbf{H}$  fields; therefore, by making the transformation [4]

$$\{\mathbf{e} = \mathbf{e}_p - \boldsymbol{\alpha}_p \cdot \mathbf{m}_p \cdot \mathbf{b}_p, \boldsymbol{\alpha} = \boldsymbol{\alpha}_p \cdot \mathbf{m}_p, \mathbf{b} = -\mathbf{m}_p \cdot \mathbf{b}_p, \mathbf{m} = \mathbf{m}_p\}, \quad (2)$$

we obtain the equivalent Tellegen representation

$$\mathbf{D}(x, y, z) = \mathbf{e} \cdot \mathbf{E}(x, y, z) + \boldsymbol{\alpha} \cdot \mathbf{H}(x, y, z), \quad (3a)$$

$$\mathbf{B}(x, y, z) = \mathbf{b} \cdot \mathbf{E}(x, y, z) + \mathbf{m} \cdot \mathbf{H}(x, y, z), \quad (3b)$$

that is more appropriate for the present purposes.

Into the source-free Maxwell curl postulates,  $\nabla \times \mathbf{E} = i\omega\mathbf{B}$  and  $-\nabla \times \mathbf{H} = i\omega\mathbf{D}$ , we substitute the spatial Fourier decompositions [14]

$$\mathbf{E}(x, y, z) = \mathbf{e}(z)e^{i\kappa_x x}e^{i\kappa_y y}, \quad (4a)$$

$$\mathbf{H}(x, y, z) = \mathbf{h}(z)e^{i\kappa_x x}e^{i\kappa_y y}, \quad (4b)$$

along with (3a) and (3b). Here,  $\kappa_x$  and  $\kappa_y$  are real numbers to be used later in order to ensure the satisfaction of Snel's laws across any  $xy$  plane, while

$$\mathbf{e}(z) = e_x(z)\mathbf{u}_x + e_y(z)\mathbf{u}_y + e_z(z)\mathbf{u}_z, \quad (5a)$$

$$\mathbf{h}(z) = h_x(z)\mathbf{u}_x + h_y(z)\mathbf{u}_y + h_z(z)\mathbf{u}_z, \quad (5b)$$

with  $\mathbf{u}_x$ ,  $\mathbf{u}_y$ , and  $\mathbf{u}_z$  being the Cartesian unit vectors.

The foregoing manipulations yield two algebraic equations and four ordinary differential equations. The main result is that we obtain the matrix differential equation

$$\{d/dz\}[f(z)] = i[P][f(z)], \quad (6a)$$

wherein

$$[f(z)] = \text{column}[e_x(z); e_y(z); h_x(z); h_y(z)] \quad (6b)$$

is a column four vector, while  $[P]$  is a  $4 \times 4$  matrix that depends on  $\mathbf{e}$ ,  $\boldsymbol{\alpha}$ ,  $\mathbf{b}$ ,  $\mathbf{m}$ ,  $\kappa_x$  and  $\kappa_y$ .

It is the eigenvectors of the matrix  $[P]$  that interest us here. As  $[P]$  is a  $4 \times 4$  matrix, we invoke four linearly independent eigenvectors

$$[\gamma_n] = \text{column}[e_{n1}; e_{n2}; h_{n1}; h_{n2}]; \quad n = 1, 2, 3, 4, \quad (7a)$$

such that

$$[P][\gamma_n] = \lambda_n[\gamma_n]; \quad n = 1, 2, 3, 4, \quad (7b)$$

where the  $\lambda_n$ 's are the corresponding eigenvalues of  $[P]$ . This invocation can be done from purely physical arguments, although mathematical justification is also possible [15]. It is not necessary that the matrix  $[P]$  have all of its eigenvalues distinct; instead, it is necessary and sufficient [16] that  $[P]$  has four linearly independent eigenvectors for it to be diagonalizable.

The solution of (6a) can be set up as [4]

$$[f(z)] = [G][L(z)][G]^{-1}[f(0)], \quad (8a)$$

where the columns of the  $4 \times 4$  matrix  $[G]$  are the eigenvectors of  $[P]$  arranged as per

$$[G] = [[\gamma_1]; [\gamma_2]; [\gamma_3]; [\gamma_4]], \quad (8b)$$

while  $[L(z)]$  is the diagonal matrix function

$$[L(z)] = \text{diagonal}[e^{i\lambda_1 z}, e^{i\lambda_2 z}, e^{i\lambda_3 z}, e^{i\lambda_4 z}]. \quad (8c)$$

Since our medium must allow wave propagation in all directions, and because the constitutive dyadics can be set down in biaxial forms, we can order the eigenvalues such that

$$\text{Real}\{\lambda_1\} > 0, \text{Real}\{\lambda_2\} > 0, \text{Real}\{\lambda_3\} < 0, \text{Real}\{\lambda_4\} < 0, \quad (9)$$

for what follows

### 3. REFLECTION AND TRANSMISSION BY A UCS

Without loss of generality, let us now consider the UCS to occupy the  $z = 0$  plane such that

$$\mathbf{u}_x \cdot \mathbf{E}(x, y, 0+) = 0, \quad (10a)$$

$$\mathbf{u}_x \cdot \mathbf{E}(x, y, 0-) = 0, \quad (10b)$$

$$\mathbf{u}_y \cdot \mathbf{E}(x, y, 0+) = \mathbf{u}_y \cdot \mathbf{E}(x, y, 0-), \quad (10c)$$

$$\mathbf{u}_y \cdot \mathbf{H}(x, y, 0+) = \mathbf{u}_y \cdot \mathbf{H}(x, y, 0-). \quad (10d)$$

Thus, the UCS is perfectly conducting along the  $x$  axis, and perfectly insulating along the  $y$  axis.

We take the incidence to take place in the lower half space (i.e.,  $z \leq 0$ ); therefore, and as a consequence of (8a), the electromagnetic fields of the Snel-compatible incident and reflected plane waves must be set up as

$$\begin{aligned} \mathbf{E}(x, y, z) = & \sum_{n=1}^4 A_n e^{i\kappa_x x} e^{i\kappa_y y} e^{i\lambda_n z} [e_{n1} \mathbf{u}_x \\ & + e_{n2} \mathbf{u}_y + e_{n3} \mathbf{u}_z]; \quad z \leq 0, \quad (11a) \end{aligned}$$

$$\begin{aligned} \mathbf{H}(x, y, z) = & \sum_{n=1}^4 A_n e^{i\kappa_x x} e^{i\kappa_y y} e^{i\lambda_n z} [h_{n1} \mathbf{u}_x \\ & + h_{n2} \mathbf{u}_y + h_{n3} \mathbf{u}_z]; \quad z \leq 0, \quad (11b) \end{aligned}$$

where  $e_{n3}$  and  $h_{n3}$  are functions of the eigenvector  $[\gamma_n]$  and the eigenvalue  $\lambda_n$  for each  $n$ . The transmitted Snel-compatible plane waves are of the form

$$\begin{aligned} \mathbf{E}(x, y, z) = & \sum_{n=1}^2 B_n e^{i\kappa_x x} e^{i\kappa_y y} e^{i\lambda_n z} [e_{n1} \mathbf{u}_x \\ & + e_{n2} \mathbf{u}_y + e_{n3} \mathbf{u}_z]; \quad z \geq 0, \quad (11c) \end{aligned}$$

$$\begin{aligned} \mathbf{H}(x, y, z) = & \sum_{n=1}^2 B_n e^{i\kappa_x x} e^{i\kappa_y y} e^{i\lambda_n z} [h_{n1} \mathbf{u}_x \\ & + h_{n2} \mathbf{u}_y + h_{n3} \mathbf{u}_z]; \quad z \geq 0, \quad (11d) \end{aligned}$$

The coefficients  $A_1$  and  $A_2$  belong to the incident,  $A_3$  and  $A_4$  to the reflected, and  $B_1$  and  $B_2$  to the transmitted plane waves.

Substitution of the field representations (11a)–(11d) into the boundary conditions (10a)–(10d) leads to the relations

$$A_3 = r_{31}[A_1 + (e_{21}/e_{11})A_2], \quad (12a)$$

$$A_4 = r_{41}[A_1 + (e_{21}/e_{11})A_2], \quad (12b)$$

for the reflected field amplitudes, and

$$B_1 = [t_{11}A_1 + t_{12}A_2], \quad (12c)$$

$$B_2 = -(e_{11}/e_{21})[t_{11}A_1 + t_{12}A_2], \quad (12d)$$

for the transmitted field amplitudes. Here,

$$\begin{aligned} \vartheta \cdot r_{31} = & e_{11}[e_{42}(e_{11}h_{22} - e_{21}h_{12}) + e_{41}(e_{22}h_{12} \\ & - e_{12}h_{22}) - h_{42}(e_{11}e_{22} - e_{21}e_{12})], \quad (13a) \end{aligned}$$

$$\begin{aligned} \vartheta \cdot r_{41} = & e_{11}[-e_{32}(e_{11}h_{22} - e_{21}h_{12}) - e_{31}(e_{22}h_{12} \\ & - e_{12}h_{22}) + h_{32}(e_{11}e_{22} - e_{21}e_{12})], \quad (13b) \end{aligned}$$

$$\begin{aligned} \vartheta \cdot t_{11} = & -e_{21}[e_{12}(e_{31}h_{42} - e_{41}h_{32}) + e_{11}(e_{42}h_{32} \\ & - e_{32}h_{42}) + h_{12}(e_{41}e_{32} - e_{31}e_{42})], \quad (13c) \end{aligned}$$

$$\begin{aligned} \vartheta \cdot t_{12} = & -e_{21}[e_{22}(e_{31}h_{42} - e_{41}h_{32}) + e_{21}(e_{42}h_{32} \\ & - e_{32}h_{42}) + h_{22}(e_{41}e_{32} - e_{31}e_{42})], \quad (13d) \end{aligned}$$

with

$$\begin{aligned} \vartheta = & (e_{11}h_{22} - e_{21}h_{12})(e_{41}e_{32} - e_{31}e_{42}) \\ & + (e_{11}e_{22} - e_{21}e_{12})(e_{31}h_{42} - e_{41}h_{32}). \quad (13e) \end{aligned}$$

The boundary value problem having been solved, we are now ready to substantiate the title of this communication. From (12a) and (12b) it is clear that the reflection amplitude ratio ( $A_3/A_4$ ) is independent of the incidence amplitude ratio ( $A_1/A_2$ ), regardless of the specific values of the incidence parameters  $\kappa_x$  and  $\kappa_y$ ; indeed, the ratio

$$\begin{aligned} (A_3/A_4) = & (r_{31}/r_{41}) \\ = & -[e_{42}(e_{11}h_{22} - e_{21}h_{12}) + e_{41}(e_{22}h_{12} - e_{12}h_{22}) \\ & - h_{42}(e_{11}e_{22} - e_{21}e_{12})]/[e_{32}(e_{11}h_{22} - e_{21}h_{12}) \\ & + e_{31}(e_{22}h_{12} - e_{12}h_{22}) - h_{32}(e_{11}e_{22} - e_{21}e_{12})] \quad (14) \end{aligned}$$

depends on  $\epsilon$ ,  $\alpha$ ,  $\beta$ ,  $m$ ,  $\kappa_x$ , and  $\kappa_y$ , but not on  $(A_1/A_2)$ . In effect, the UCS acts as a Brewster screen [4, 10].

There is more to relate yet. Equations (12c) and (12d) tell us that the transmission amplitude ratio  $(B_1/B_2)$  is also independent of the incidence amplitude ratio  $(A_1/A_2)$ , regardless of the specific values of the incidence parameters  $\kappa_x$  and  $\kappa_y$ , because

$$(B_1/B_2) = -(e_{21}/e_{11}). \quad (15)$$

This ratio, in fact, follows directly from the boundary condition (10b).

The premise of this communication now stands proved: The reflection ratio  $(A_3/A_4)$  as well as the transmission ratio  $(B_1/B_2)$  do not depend on the incidence amplitude ratio  $(A_1/A_2)$  for any given values of  $\kappa_x$  and  $\kappa_y$ . Since the reflected and the transmitted polarization states are thus independent of the incidence polarization state, a UCS is a super-Brewster screen. We must take note that a UCS will act in this fashion regardless of the nature of electromagnetic medium it is embedded in, provided the medium is nondiffusive, linear, and homogeneous.

## REFERENCES

1. G. Toraldo di Francia, "Electromagnetic Cross Section of a Small Circular Disc with Directional Conductivity," *Nuovo Cimento*, Vol. 3, 1956, pp. 1276-1284.
2. D. B. Brewster, "On the Laws which Regulate the Polarization of Light by Reflection from Transparent Bodies," *Philos. Trans. R. Soc. London*, Vol. 105, 1815, pp. 105-129.
3. R. W. Wood, *Physical Optics*, Optical Society of America, Washington, DC, 1988.
4. A. Lakhtakia, "General Schema for the Brewster Conditions," *Optik*, Vol. 90, 1992, pp. 184-186.
5. S. N. Karp and F. C. Karal, "Excitation of Surface Waves on a Unidirectionally Conducting Screen by a Phased Line Source," *IEEE Trans. Antennas Propagat.*, Vol. AP-12, 1964, pp. 470-478.
6. S. R. Seshadri, "Plane Wave Reflection and Transmission by a Unidirectionally Conducting Screen," *Proc. IEEE*, Vol. 57, 1969, pp. 722-724.
7. S. R. Seshadri, "Polarization Control by a Twist Reflector," *J. Appl. Phys.*, Vol. 70, 1991, 1952-1956.
8. J. C. Monzon, "Scattering by a Moving Unidirectionally Conducting Screen," *IEEE Trans. Antennas Propagat.*, Vol. AP-37, 1989, pp. 64-70.
9. J. C. Monzon, "Guidance of Waves by a Moving Unidirectionally Conducting Screen," *IEEE Trans. Antennas Propagat.*, Vol. 37, 1989, pp. 1291-1300.
10. A. Lakhtakia, "Plane Wave Scattering Response of a Unidirectionally Conducting Screen Immersed in a Bi-Isotropic Medium," *Microwave Opt. Technol. Lett.*, Vol. 5, 1992, pp. 163-166; Errata, Vol. 5, 1992, p. 476.
11. E. J. Post, *Formal Structure of Electromagnetics*, North-Holland, Amsterdam, 1962.
12. P. S. Reese and A. Lakhtakia, "Low-Frequency Electromagnetic Properties of an Alternating Stack of Thin Uniaxial Dielectric Laminae and Uniaxial Magnetic Laminae," *Z. Naturforsch. Teil A*, Vol. 46, 1991, pp. 384-388.
13. J. L. Tsalamengas, "Interaction of Electromagnetic Waves with General Bianisotropic Slabs," *IEEE Trans. Antennas Propagat.*, Vol. AP-40, 1992, pp. 1870-1878.
14. A. Lakhtakia, "Cartesian Solutions of Maxwell's Equations for Linear, Anisotropic Media—Extension of the Mrozowski Algorithm," *Arch. Elektron. Über.*, Vol. 41, 1987, pp. 178-179.

15. P. Hillion, personal communication, 1992.

16. P. K. Chattopadhyay, *Mathematical Physics*, Wiley Eastern, New Delhi, 1990.

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## ON THE DESIGN AND ANALYSIS OF ULTRABROADBAND ANTENNA WINDOWS

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### KEY TERMS

*A-sandwich window, C-sandwich window, optical ray tracing, power transmission coefficient*

### ABSTRACT

The design, analysis, and the results obtained on two ultrabroadband antenna windows, operating over 2-18 GHz, one in A-sandwich and the other in C-sandwich construction are presented. Curves are given for theoretical and practical power transmission coefficients. The maximum transmission loss measured in the A-sandwich window is <1 dB and that of the C-sandwich window is <1.5 dB over the complete band. © 1993 John Wiley & Sons, Inc.

### 1. INTRODUCTION

Modern electronic warfare systems demand antennas operating over multioctave frequency bands with high direction-finding accuracy. For many applications the antenna will be enclosed by an antenna window (radome) and the overall system performance will be critically dependent upon the characteristics of this electromagnetic window. The radome design presents many challenges and is an interdisciplinary problem involving several fields, such as electrical, mechanical, thermal, environmental, etc. The design problem will become more stringent when the antenna windows have to operate over an ultrabroadband of frequencies, say, 2-18 GHz. This article describes the design philosophy, analysis, and the test results obtained on two ultrabroadband antenna windows operating over a frequency band of 2-18 GHz, one in an A-sandwich and the other in a C-sandwich type of wall construction.

### 2. DESIGN PHILOSOPHY

An electrically thin wall window (thickness  $< \lambda/20$ ) is better from the electromagnetic transparency point of view, but it suffers due to the poor mechanical strength. For high mechanical strength the window should have a thickness of odd multiples of  $\lambda/4$ , but then the problem is narrow bandwidth. Sometimes it becomes inevitable to the radome designer to accept a trade-off to meet the given electrical and mechanical specifications.

In order to have both high strength and ultrabroadband characteristics, the sandwich type of wall construction [1-3] is employed in the antenna window designs. Sandwich construction can be of type A, B, C, or in general multiple layer