Brewster Condition for Planar Interfaces of Natural Optically Active Media

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Reflection of planewaves at the planar interface of two natural optically active media has been examined in order to obtain the Brewster condition.

Sir David Brewster published [1] in 1815 the results of his experiments on the reflection of unpolarized light from planar dielectric-dielectric interfaces; his work gave rise to the Brewster angle and was condensed by him into the *Brewster law*: If unpolarized light is incident under this angle, the reflected light is plane-polarized. During the 1950s, however, it appears [2] that the original definition of the Brewster angle as the polarizing angle was superseded by a new one: a *zero-reflection* angle. In my opinion, the increasing use of complex (but homogeneous) materials strongly suggests a return to the original definition, and even a broadening of the concept behind the Brewster angle.

Plane waves propagating towards (or away from) a planar interface can be expressed in terms of two distinct and orthogonal eigenmodes [3]. It is conjectured that a condition may exist when the ratio of the amplitudes of the two eigenmodes of the reflected wave is independent of the ratio of the amplitudes of the two eigenmodes of the incident wave.

This condition, to be called after Brewster, may be easily quantified in terms of the horizontal wavenumber $\varkappa$ that comes in as a consequence of Snel's laws. For the fulfilment of the Brewster condition, the value of the wavenumber $\varkappa$ depends on the relative properties of the two homogeneous media occupying either side of the planar interface. This conjecture has been tested for the planar interfaces of (i) a natural optically active [4] and an isotropic dielectric-magnetic medium [5], (ii) a natural optically active and an uniaxial dielectric medium [6], and (iii) an isotropic dielectric-magnetic and a general uniaxial medium [7]. In this brief communication, the conjecture will be shown to hold true for the planar interfaces of two natural optically active media (chiral) also.

Consider the material interface $z=0$. The half-space $z \geq 0$ is filled with the homogeneous, chiral medium characterized by

$$\mathbf{D} = \varepsilon_a \mathbf{E} + \varepsilon_a \beta_a \nabla \times \mathbf{E}, \quad \mathbf{B} = \mu_a \mathbf{H} + \mu_a \beta_a \nabla + \mathbf{H};$$

while the half-space $z \leq 0$ is filled with another chiral medium $[\mathbf{D} = \varepsilon_b \mathbf{E} + \varepsilon_b \beta_b \nabla \times \mathbf{E}, \mathbf{B} = \mu_b \mathbf{H} + \mu_b \beta_b \nabla \times \mathbf{H}]$. An $\exp[-i\omega t]$ time-dependence is implicit thoroughout; the wavenumbers

$$\gamma_{1a} = k_a/(1-k_a\beta_a), \quad \gamma_{2a} = k_a/(1+k_a\beta_a),$$
$$\gamma_{1b} = k_b/(1-k_b\beta_b), \quad \gamma_{2b} = k_b/(1+k_b\beta_b)$$

are defined for the two media, with $k_{a,b} = \omega\sqrt{(\mu_{a,b}\,\varepsilon_{a,b})}$ being shorthand notations; and the impedances are given by $\eta_{a,b} = \sqrt{(\mu_{a,b}/\varepsilon_{a,b})}$.

In a natural optically active medium, the electromagnetic fields, **E** and **H**, can be expressed in terms of Beltrami fields $\mathbf{Q}_1$ and $\mathbf{Q}_2$ [4, 8]; thus,

$$\mathbf{E}_{a,b} = \mathbf{Q}_{1a,b} - i\eta_{a,b}\,\mathbf{Q}_{2a,b} \quad \text{and}$$
$$\mathbf{H}_{a,b} = \mathbf{Q}_{2a,b} - (i/\eta_{a,b})\,\mathbf{Q}_{1a,b}$$

in the two regions. Without loss of generality, let $z \geq 0$ be the zone of incidence and reflection, while the zone $z \leq 0$ be that of transmission. Consequently the planewave representation for the two chiral media can be set up as [5, 6]

$$\mathbf{Q}_{1a} = A_1 [\mathbf{e}_y + i(\alpha_{1a}\mathbf{e}_x + \varkappa\,\mathbf{e}_z)/\gamma_{1a}] \exp[i(\varkappa x - \alpha_{1a} z)] + R_1 [\mathbf{e}_y + i(-\alpha_{1a}\mathbf{e}_x + \varkappa\,\mathbf{e}_z)/\gamma_{1a}] \exp[i(\varkappa x + \alpha_{1a} z)]; \quad z \geq 0, \tag{1a}$$

$$\mathbf{Q}_{2a} = A_2 [\mathbf{e}_y - i(\alpha_{2a}\mathbf{e}_x + \varkappa\,\mathbf{e}_z)/\gamma_{2a}] \exp[i(\varkappa x - \alpha_{2a} z)] + R_2 [\mathbf{e}_y - i(-\alpha_{2a}\mathbf{e}_x + \varkappa\,\mathbf{e}_z)/\gamma_{2a}] \exp[i(\varkappa x + \alpha_{2a} z)]; \quad z \geq 0, \tag{1b}$$

$$\mathbf{Q}_{1b} = T_1 [\mathbf{e}_y + i(\alpha_{1b}\mathbf{e}_x + \varkappa\,\mathbf{e}_z)/\gamma_{1b}] \exp[i(\varkappa x - \alpha_{1b} z)]; \quad z \leq 0, \tag{1c}$$

$$\mathbf{Q}_{2b} = T_2 [\mathbf{e}_y - i(\alpha_{2b}\mathbf{e}_x + \varkappa\,\mathbf{e}_z)/\gamma_{2b}] \exp[i(\varkappa x - \alpha_{2b} z)]; \quad z \leq 0, \tag{1d}$$

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The coefficients A_1 and A_2 are the complex amplitudes of the planewave eigenmodes incident on the interface, R_1 and R_2, of the planewave eigenmodes reflected off the interface into the zone $z \geq 0$; while T_1 and T_2 are for the planewave eigenmodes transmitted into the zone $z \leq 0$. Finally, $\varkappa$ is the horizontal wavenumber required by Snel's law to satisfy the phase-matching condition of the interface $z=0$;

$$\alpha_{1a,b} = +\sqrt{(\gamma_{1a,b}^2 - \varkappa^2)} \quad \text{and}$$
$$\alpha_{2a,b} = +\sqrt{(\gamma_{2a,b}^2 - \varkappa^2)},$$

and $\mathbf{e}_x$, $\mathbf{e}_y$, and $\mathbf{e}_z$ are the unit Cartesian vectors.

The boundary value problem is solved by ensuring the continuity of the tangential components of the **E** and the **H** fields across the interface $z=0$. For a given $\varkappa$, the resulting solution is best stated in matrix notation as follows:

$$\begin{pmatrix} R_1 \\ R_2 \end{pmatrix} = \begin{pmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}, \quad \begin{pmatrix} T_1 \\ T_2 \end{pmatrix} = \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}. \tag{2a, b}$$

The various Fresnel reflection and transmission coefficients involved in the foregoing matrices are given as

$$r_{11} = (u_- + v_-)/(u_+ + v_+), \tag{3a}$$
$$r_{12} = 2w\,\xi_{2a}(i\,\eta_a)/(u_+ + v_+), \tag{3b}$$
$$r_{12} = 2w\,\xi_{1a}/(i\,\eta_a)\,(u_+ + v_+), \tag{3c}$$
$$r_{22} = (u_- - v_-)/(u_+ + v_+), \tag{3d}$$
$$t_{11} = 4\eta_b(\eta_a + \eta_b)\,\xi_{1b}(\xi_{2a} + \xi_{2b})/(u_+ + v_+), \tag{4a}$$
$$t_{12} = -4i\,\eta_a\,\eta_b(\eta_a - \eta_b)\,\xi_{2a}(\xi_{1a} - \xi_{1b})/(u_+ + v_+), \tag{4b}$$
$$t_{21} = 4i(\eta_a - \eta_b)\,\xi_{1a}(\xi_{2a} - \xi_{2b})/(u_+ + v_+), \tag{4c}$$
$$t_{22} = 4\eta_a(\eta_a + \eta_b)\,\xi_{2a}(\xi_{1a} + \xi_{1b})/(u_+ + v_+). \tag{4d}$$

The following functions have been used in (3a–d) and (4a–d):

$$u_\pm = 4\eta_a\eta_b(\xi_{1a}\,\xi_{2a} \pm \xi_{1b}\,\xi_{2b}), \tag{5a}$$
$$w = (\eta_a^2 - \eta_b^2)(\xi_{1b} + \xi_{2b}), \tag{5b}$$
$$v_\pm = (\eta_a - \eta_b)^2(\xi_{1a}\,\xi_{1b} \pm \xi_{2a}\,\xi_{2b}) + (\eta_a + \eta_b)^2(\xi_{1a}\,\xi_{2b} \pm \xi_{2a}\,\xi_{1b}), \tag{5c}$$
$$\xi_{1a,b} = \alpha_{1a,b}/\gamma_{1a,b}, \tag{5d}$$
$$\xi_{2a,b} = \alpha_{2a,b}/\gamma_{2a,b}. \tag{5e}$$

In order that the reflection amplitude ratio (R_1/R_2) be independent of the incidence amplitude ratio (A_1/A_2), the equality

$$r_{12}r_{21} = r_{11}r_{22} \tag{6}$$

must be satisfied. For the condition (6) to hold, $\varkappa$ must satisfy the relation $u_+ = v_+$, i.e.,

$$4\eta_a\,\eta_b(\xi_{1a}\,\xi_{2a} + \xi_{1b}\,\xi_{2b}) = (\eta_a - \eta_b)^2(\xi_{1a}\,\xi_{1b} + \xi_{2a}\,\xi_{2b}) + (\eta_a + \eta_b)^2(\xi_{1a}\,\xi_{2b} + \xi_{2a}\,\xi_{1b}). \tag{7}$$

It is to be noted that interchanging the symbols a and b in the subscripts of the quantities appearing in (7) does not alter that equation; ergo, (7) broadens the concept of the Brewster law for planar interfaces between two natural optically active materials, regardless of which half-space the incidence is from. Hence, (7) should be called the Brewster condition for chiral-chiral interfaces.

The conjecture mentioned at the beginning of this communication, and verified in three earlier instances [5–7], thus holds true for yet another case.

[1] D. B. Brewster, Philos. Trans. Roy. Soc. London **105,** 105 (1815).
[2] A. Lakhtakia, OSA Optics News **15**(6), 14 (1989).
[3] H. C. Chen, Theory of Electromagnetic Waves, Wiley, New York 1983.
[4] A. Lakhtakia, V. K. Varadan, and V. V. Varadan, Time-Harmonic Electromagnetic Fields in Chiral Media, Springer-Verlag, Berlin 1989.
[5] A. Lakhtakia, V. V. Varadan, and V. K. Varadan, J. Opt. Soc. Amer. A **7,** 1654 (1990).
[6] A. Lakhtakia, Optik **84,** 160 (1990).
[7] A. Lakhtakia, K. V. Varadan, and V. V. Varadan, J. Mod. Opt. **38,** 649 (1991).
[8] A. Lakhtakia, Speculat. Sci. Tech. **14,** 2 (1991).