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RECIPROCITY AND THE CONCEPT OF THE BREWSTER WAVENUMBER

Akhlesh Lakhtakia and Jeffrey R. Diamond

Department of Engineering Science and Mechanics
The Pennsylvania State University
University Park, Pennsylvania 16802-1484

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Reflection of planewaves at the planar interface of two biisotropic media has been examined in order to obtain the Brewster wavenumber. This examination shows that the Brewster wavenumber is, at least, not necessarily affected by the reciprocity or the non-reciprocity of the media on either side of the planar interface.

1. The Brewster Wavenumber Concept

Roughly around the end of the Second World War, the mode of presenting the Brewster angle in textbooks underwent a drastic change. As has been cataloged in [1], the original definition as a polarizing angle was replaced by that of a zero-reflection angle. This new definition, being related to a pathological condition of a Fresnel coefficient for a vacuum/dielectric interface [2], is certainly the easier to remember, but represents just one among the many other interesting phenomena that can take place at such interfaces [3].

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The original definition, given by Brewster himself, is far more exciting in that it can be profitably extended to other bimaterial interfaces. In this connection, the concept of a Brewster wavenumber has been put forth at Penn State during the past few years. It is the objective of this communication to shed further light on this concept.

Plane waves propagating towards (or away from) a planar interface between two homogenous regions can be expressed in terms of two distinct and orthogonal eigenmodes in either region [4]. Based on an extension of Brewster's empirical deductions, it has been conjectured that a condition may exist when the ratio of the amplitudes of the two eigenmodes of the reflected field is independent of the ratio of the amplitudes of the two eigenmodes of the incident field. This condition may be easily quantified in terms of the horizontal wavenumber κ that comes in as a consequence of Snel's laws; a horizontal wavenumber fulfiling this condition may be termed as the Brewster wavenumber, its value depending, in general, on the frequency as well as on the properties of the two homogeneous media occupying either side of the planar interface. This conjecture has been tested for the planar interfaces of (i) a natural optically active [5] and an isotropic dielectric-magnetic media [6], (ii) a natural optically active and an uniaxial dielectric media [7], and (iii) an isotropic dielectric-magnetic and a general uniaxial media [8].

The cases examined heretofore [6-8] involved media that are reciprocal [9]. Of great interest, therefore, is the possible effect of nonreciprocity on the Brewster wavenumber concept. In order to explore this, we turn our attention to Fedorov biisotropic media [10, 11] that are the nonreciprocal generalizations of the isotropic natural optically active media [5, 12].

2. Fedorov Biisotropic Medium

A Fedorov biisotropic medium can be characterized by the frequency-dependent $\{\exp(-i\omega t)\}$ constitutive equations

$$\mathbf{D} = \varepsilon\,\mathbf{E} + \varepsilon\alpha\,\nabla\times\mathbf{E}, \tag{1a}$$
$$\mathbf{B} = \mu\,\mathbf{H} + \mu\beta\,\nabla\times\mathbf{H}, \tag{1b}$$

in which the pseudoscalar parameters α and β carry the dimension of length. We define the quantities $k = \omega\surd(\varepsilon\mu)$ and $\eta = \surd(\mu/\varepsilon)$ as usual, but noting that k is not a wavenumber and η is not an intrinsic impedance here.

Application of a diagonalizing transform [5, 13] yields the field decomposition

$$\mathbf{E} = \mathbf{Q}_1 + \mathbf{Q}_2, \tag{2a}$$
$$\mathbf{H} = i[\mathbf{Q}_1/\eta_1 + \mathbf{Q}_2/\eta_2], \tag{2b}$$

where the Beltrami fields [14, 15] satisfy the circulation conditions

$$\nabla\times\mathbf{Q}_1 = \gamma_1\,\mathbf{Q}_1, \tag{3a}$$
$$\nabla\times\mathbf{Q}_2 = -\,\gamma_2\,\mathbf{Q}_2. \tag{3b}$$

It is evident that such a medium is birefringent, with the two wavenumbers given by [13]

$$\gamma_1 = k\,[1 - k^2\alpha\beta]^{-1}\,\{\surd[1 + k^2(\alpha-\beta)^2/4] + k(\alpha+\beta)/2\}, \tag{4a}$$
$$\gamma_2 = k\,[1 - k^2\alpha\beta]^{-1}\,\{\surd[1 + k^2(\alpha-\beta)^2/4] - k(\alpha+\beta)/2\}, \tag{4b}$$

while the corresponding impedances are given by

$$\eta_1 = -\,\eta/\{\surd[1 + k^2(\alpha-\beta)^2/4] + k(\alpha-\beta)/2\}, \tag{5a}$$
$$\eta_2 = \eta\{\surd[1 + k^2(\alpha-\beta)^2/4] + k(\alpha-\beta)/2\}. \tag{5b}$$

We note in passing that $\gamma_1\gamma_2 = k^2\,[1 - k^2\alpha\beta]^{-1}$ and $\eta_1\eta_2 = -\,\eta^2$. Furthermore, for the Fedorov media to be reciprocal, it is necessary that $\alpha = \beta$ [5]; in that case, $\gamma_1 = k/[1 - k\beta]$, $\gamma_2 = k/[1 + k\beta]$, $\eta_1 = -\,\eta$ and $\eta_2 = \eta$.

3. Fresnel Coefficients

Consider now the bimaterial interface z = 0. The half-space $z \geq 0$ is filled with the homogeneous Fedorov medium characterized by $\mathbf{D} = \varepsilon_a \mathbf{E} + \varepsilon_a \alpha_a \nabla\times\mathbf{E}$, $\mathbf{B} = \mu_a \mathbf{H} + \mu_a \beta_a \nabla\times\mathbf{H}$; while the half-space $z \leq 0$ is filled with another Fedorov medium [$\mathbf{D} = \varepsilon_b \mathbf{E} + \varepsilon_b \alpha_b \nabla\times\mathbf{E}$, $\mathbf{B} = \mu_b \mathbf{H} + \mu_b \beta_b \nabla\times\mathbf{H}$]. The wavenumbers ($\gamma_{1a,b}$ and $\gamma_{2a,b}$) and the impedances ($\eta_{1a,b}$ and $\eta_{2a,b}$) for both media are defined as in Section 2.

Without loss of generality, let $z \geq 0$ be the zone of incidence and reflection, while the zone $z \leq 0$ be that of transmission. Consequently the planewave representation for the two media can be set up as [6, 7, 16]

$$\mathbf{Q}_{1a} = A_1[\mathbf{e}_y + i(\sigma_{1a}\mathbf{e}_x + \kappa\mathbf{e}_z)/\gamma_{1a}]\exp[i(\kappa x - \sigma_{1a} z)] + R_1[\mathbf{e}_y + i(-\sigma_{1a}\mathbf{e}_x + \kappa\mathbf{e}_z)/\gamma_{1a}]\exp[i(\kappa x + \sigma_{1a} z)]\,;\; z \geq 0, \quad (6a)$$

$$\mathbf{Q}_{2a} = A_2[\mathbf{e}_y - i(\sigma_{2a}\mathbf{e}_x + \kappa\mathbf{e}_z)/\gamma_{2a}]\exp[i(\kappa x - \sigma_{2a} z)] + R_2[\mathbf{e}_y - i(-\sigma_{2a}\mathbf{e}_x + \kappa\mathbf{e}_z)/\gamma_{2a}]\exp[i(\kappa x + \sigma_{2a} z)]\,;\; z \geq 0. \quad (6b)$$

$$\mathbf{Q}_{1b} = T_1[\mathbf{e}_y + i(\sigma_{1b}\mathbf{e}_x + \kappa\mathbf{e}_z)/\gamma_{1b}]\exp[i(\kappa x - \sigma_{1b} z)]\,;\; z \leq 0, \quad (6c)$$

$$\mathbf{Q}_{2b} = T_2[\mathbf{e}_y - i(\sigma_{2b}\mathbf{e}_x + \kappa\mathbf{e}_z)/\gamma_{2b}]\exp[i(\kappa x - \sigma_{2b} z)]\,;\; z \leq 0. \quad (6d)$$

The coefficients A_1 and A_2 are the complex amplitudes of the planewave eigenmodes incident on the interface; R_1 and R_2, of the planewave eigenmodes reflected off the interface into the zone $z \geq 0$; while T_1 and T_2 are for the planewave eigenmodes transmitted into the zone $z \leq 0$. Finally, κ is the horizontal wavenumber required by Snel's law to satisfy the phase-matching condition of the interface z = 0; $\sigma_{1a,b} = +\sqrt{(\gamma_{1a,b}{}^2 - \kappa^2)}$ and $\sigma_{2a,b} = +\sqrt{(\gamma_{2a,b}{}^2 - \kappa^2)}$; and $\mathbf{e}_x$, $\mathbf{e}_y$ and $\mathbf{e}_z$ are the unit Cartesian vectors.

The boundary value problem is solved by ensuring the continuity of the tangential components of the **E** and the **H** fields across the interface $z = 0$. For a given κ, the resulting solution can be stated as follows:

$$R_1 = r_{11} A_1 + r_{12} A_2, \quad (7a)$$
$$R_2 = r_{21} A_1 + r_{22} A_2, \quad (7b)$$

and

$$T_1 = t_{11} A_1 + t_{12} A_2, \quad (8a)$$
$$T_2 = t_{21} A_1 + t_{22} A_2. \quad (8b)$$

The four Fresnel reflection coefficients involved in (7a,b) can be specified as

$$\begin{aligned}\Delta \bullet r_{11} = &(\eta_{1a} - \eta_{1b})(\eta_{2a} - \eta_{2b})(\zeta_{1a}\zeta_{1b} - \zeta_{2a}\zeta_{2b}) \\ &- (\eta_{1a} - \eta_{2a})(\eta_{1b} - \eta_{2b})(\zeta_{1a}\zeta_{2a} - \zeta_{1b}\zeta_{2b}) \\ &- (\eta_{1a} - \eta_{2b})(\eta_{1b} - \eta_{2a})(\zeta_{1a}\zeta_{2b} - \zeta_{1b}\zeta_{2a}), \quad (9a)\end{aligned}$$

$$\begin{aligned}\Delta \bullet r_{22} = &- (\eta_{1a} - \eta_{1b})(\eta_{2a} - \eta_{2b})(\zeta_{1a}\zeta_{1b} - \zeta_{2a}\zeta_{2b}) \\ &- (\eta_{1a} - \eta_{2a})(\eta_{1b} - \eta_{2b})(\zeta_{1a}\zeta_{2a} - \zeta_{1b}\zeta_{2b}) \\ &+ (\eta_{1a} - \eta_{2b})(\eta_{1b} - \eta_{2a})(\zeta_{1a}\zeta_{2b} - \zeta_{1b}\zeta_{2a}), \quad (9b)\end{aligned}$$

$$\Delta \bullet r_{12} = -2\,(\eta_{1a}/\eta_{2a})(\eta_{2a} - \eta_{1b})(\eta_{2a} - \eta_{2b})\,\zeta_{2a}(\zeta_{1b} + \zeta_{2b}), \quad (9c)$$

and

$$\Delta \bullet r_{21} = -2\,(\eta_{2a}/\eta_{1a})(\eta_{1a} - \eta_{1b})(\eta_{1a} - \eta_{2b})\,\zeta_{1a}(\zeta_{1b} + \zeta_{2b}), \quad (9d)$$

where

$$\begin{aligned}\Delta = &(\eta_{1a} - \eta_{1b})(\eta_{2a} - \eta_{2b})(\zeta_{1a}\zeta_{1b} + \zeta_{2a}\zeta_{2b}) \\ &- (\eta_{1a} - \eta_{2a})(\eta_{1b} - \eta_{2b})(\zeta_{1a}\zeta_{2a} + \zeta_{1b}\zeta_{2b}) \\ &- (\eta_{1a} - \eta_{2b})(\eta_{1b} - \eta_{2a})(\zeta_{1a}\zeta_{2b} + \zeta_{1b}\zeta_{2a}), \quad (10)\end{aligned}$$

$\zeta_{1a,b} = \sigma_{1a,b}/\gamma_{1a,b}$ and $\zeta_{2a,b} = \sigma_{2a,b}/\gamma_{2a,b}$. Similarly, the transmission coefficients of (8a,b) can be calculated from

$$\Delta \bullet t_{11} = -2\,(\eta_{1b}/\eta_{1a})\,(\eta_{1a}-\eta_{2a})\,(\eta_{1a}-\eta_{2b})\,\zeta_{1a}\,(\zeta_{2a}+\zeta_{2b}), \tag{11a}$$

$$\Delta \bullet t_{12} = -2\,(\eta_{1b}/\eta_{2a})\,(\eta_{1a}-\eta_{2a})\,(\eta_{2a}-\eta_{2b})\,\zeta_{2a}\,(\zeta_{1a}-\zeta_{2b}), \tag{11b}$$

$$\Delta \bullet t_{21} = 2\,(\eta_{2b}/\eta_{1a})\,(\eta_{1a}-\eta_{2a})\,(\eta_{1a}-\eta_{1b})\,\zeta_{1a}\,(\zeta_{2a}-\zeta_{1b}), \tag{11c}$$

and

$$\Delta \bullet t_{22} = 2\,(\eta_{2b}/\eta_{2a})\,(\eta_{1a}-\eta_{2a})\,(\eta_{2a}-\eta_{1b})\,\zeta_{2a}\,(\zeta_{1a}+\zeta_{1b}). \tag{11d}$$

The specific manner of presenting the reflection and the transmission coefficients permits one to observe the interplay of the four impedances, and that of the four wavenumbers, at the bimaterial interface.

4. The Brewster Wavenumber

In order that the reflection amplitude ratio (R_1/R_2) be independent of the incidence amplitude ratio (A_1/A_2), the equality

$$r_{12}\,r_{21} = r_{11}\,r_{22} \tag{12}$$

must be satisfied. For (12) to hold, the horizontal wavenumber κ must be the solution of the equation

$$\begin{aligned}(\eta_{1a}-\eta_{1b})\,(\eta_{2a}-\eta_{2b})\,(\zeta_{1a}\zeta_{1b}+\zeta_{2a}\zeta_{2b}) \\ +\,(\eta_{1a}-\eta_{2a})\,(\eta_{1b}-\eta_{2b})\,(\zeta_{1a}\zeta_{2a}+\zeta_{1b}\zeta_{2b}) \\ =(\eta_{1a}-\eta_{2b})\,(\eta_{1b}-\eta_{2a})\,(\zeta_{1a}\zeta_{2b}+\zeta_{1b}\zeta_{2a}).\end{aligned} \tag{13}$$

It is to be noted that interchanging the symbols *a* and *b* in the subscripts of the quantities appearing in (13) does not alter that equation; ergo, (13) broadens the concept of the Brewster angle, regardless of which half-space the incidence is from. Hence, (13) should be called the Brewster condition for the interfaces under consideration here, and the particular value of κ satisfying this condition should be termed the Brewster wavenumber.

Specifically, it is to be noted that the two media considered here are nonreciprocal; yet, the Brewster wavenumber concept holds as firmly as in the previously considered cases [6-8] involving reciprocal media. This examination therefore demonstrates that the Brewster wavenumber is, at least, *not necessarily* affected by the reciprocity or the non-reciprocity of the media on either side of the planar interface.

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