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Diffusion in electromagnetic theory

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In a semiconductor, charge is transported by conduction as well as by diffusion. Here the effect of diffusion is examined by incorporating diffusion in the constitutive equations. Infinite-medium Green's functions and mathematical statements of the Huygens's principle are derived. Conservation of energy, conservation of charge, and the creation of screen potentials and induced charges due to diffusion are examined.

1 Introduction

In an intrinsic semiconductor the charge is carried by electrons as well as by electron-vacancies known as holes[1]. Enhancement of current mechanisms is affected by doping the intrinsic semiconductor with electron-rich (or electron-poor) atoms, giving rise to a semiconductor in which the majority carriers are electrons (or holes) and the minority carriers are holes (or electrons). Generation of electron-hole pairs is constantly taking place: if the minority number density suddenly decreases below the equilibrium value, then thermal generation causes it to increase exponentially back to equilibrium. Conversely, if the minority carrier number density suddenly increases above an equilibrium value, then it decays exponentially down to the equilibrium value because of recombination with the majority species. In the presence of an applied static electric field, however, a dc bias drift current is created, which equals the carrier charge times the carrier drift velocity. This endows the material with anisotropic characteristics, since the dc bias creates a preferred direction. Furthermore, it also makes the relevant equations non-linear, which are however generally linearized for analysis[2].

Here we shall not be concerned with biased semiconductors however, because our modest aim is to elucidate some basic characteristics of the diffusion process. For the sake of simplicity, we shall ignore generation and recombination, so that the time scales are macroscopic in general. Also, we will only consider single carrier transport; in particular, the single carriers will be electrons. This idealized situation is actually a good small-signal model for a highly doped, uniform, n-type semiconductor.

Before carrying on, we note that the classical electromagnetic theory of semiconductors has received only scant attention, and that too because of interest in utilizing semiconductors as substrates for microstrip circuits. Some work has been reported by Sumi[3], which was further enhanced by Zotter[4]. Due to Davis and Krowne[5], a more comprehensive analysis for planar semiconductor-free space interfaces has recently become available. It is expected that the sequel may also be of interest in the investigation of plasmons[6,7].

2 The Diffusion Process

Let $\mathbf{J}$ be the volume carrier current density, and ρ be the volume carrier charge (electron) density. Then, the conservation of charge implies that

$$\nabla \cdot \mathbf{J} = -\partial\rho/\partial t \qquad \ldots(1)$$

On the other hand, Fick's law of diffusion would give us

$$\mathbf{J} = -D_n \nabla\rho \qquad \ldots(2)$$

where D_n is the diffusion constant for electrons in the n-type material. Combining Eqs (1) and (2) leads to the diffusion equation

$$\partial\rho/\partial t = D_n \nabla^2\rho \qquad \ldots(3)$$

which is not symmetric under time-reversal, thereby indicative of a one-way (irreversible) process. Also, Eq. (3) suggests that an arbitrary charge configuration, confined within some finite volume, will decay away with a time constant

$$\tau = L^2/D_n \qquad \ldots(4)$$

L being a length characteristic of the spatial variations of the charge in the material.

Eq. (3) is a statement of charge conservation. As such, it should be consistent with Maxwell's equations. With ε_s and μ_s being, respectively, the (real) permittivity and permeability of the semiconductor, it is easily seen that the sourceless Faraday and Ampere-Maxwell equations can read

$$\nabla \times \mathbf{E} = -\mu_s \partial \mathbf{H}/\partial t;$$

$$\nabla \times \mathbf{H} = \varepsilon_s \partial \mathbf{E}/\partial t - \varepsilon_s D_n \nabla\nabla \cdot \mathbf{E} \qquad \ldots(5a,b)$$

And Eq. (5b) leads to Eq. (3), since Gauss' law would have

$$\nabla \cdot (\varepsilon_s \mathbf{E}) = \rho \qquad \ldots(5c)$$

In view of the above, it is not surprising why an RLC transmission line is described by equations similar to Eqs (1) and (2), with the resistance R clearly playing the role of the reciprocal of a diffusion constant. The linkage of wave and "diffusive" processes in RLC transmission lines has been known for some time. It has been exploited in some complex applications, such as for studying lightning where the corona involves a charge diffusion process[8].

No conduction current was added here on the right side of Eq. (5b), in contrast to Davis and Krowne[5]. This is in order to emphasize the fact that conductivity as a measure of carrier collisions shares attributes common with several other transport phenomena. Examples include (i) diffusion theory; (ii) heat conduction, a very lossy wave process involving energy transfer at a very high frequency; and (iii) viscosity or frictional force, where momentum transport can either be ordinary or magnetic as found in magnetohydrodynamics. These are lossy processes which result in highly distorted and attenuated fields with virtually no time delay.

Inclusion of the conducting terms $\sigma\mathbf{E}$ on the right side of Eq. (5b), and taking the divergence of the resulting equation, reveals that the charge equation retains its diffusive character in the form

$$\{\partial/\partial t\}[\rho \exp(\sigma t/\varepsilon_s)] = D_n \nabla^2 [\rho \exp(\sigma t/\varepsilon_s)] \qquad \ldots(6)$$

as may be observed by comparing Eq. (3) with Eq. (6). This implies that conductivity as well as diffusion contribute to the damping of the oscillations. On the other hand, inclusion of conductivity results in the following (**D**-independent) equation for the magnetic field lines:

$$\sigma\mu_s \partial \mathbf{H}/\partial t - \nabla^2 \mathbf{H} = \varepsilon_s \mu_s \{\partial^2 \mathbf{H}/\partial t^2\} \qquad \ldots(7)$$

as is customary when dealing with conducting fluids[9]. The left side of Eq. (7) suggests that the field tends to decay with a time constant

$$\tau_1 = \sigma\mu_s L_1^2 \qquad \ldots(8)$$

where L_1 is a characteristic length of the spatial variations of **H**. At very low frequencies, the right side of Eq. (7) can be neglected to yield a diffusion equation

$$\sigma\mu_s \partial \mathbf{H}/\partial t = \nabla^2 \mathbf{H} \qquad \ldots(9)$$

Note must be made of the resemblance of Eq. (9) to the usual approximation in magnetohydrodynamics[9]. We conclude that Eq. (7) represents a wave with time constant $\tau_2 = \sqrt{(\varepsilon_s \mu_s)} L_1$ which is competing against the damped diffusion of field lines characterised by the time constant τ_1. When $\tau_2 \gg \tau_1$, diffusion dominates; whereas for $\tau_1 \gg \tau_2$, the wave nature dominates.

From the above mentioned discussion, we conclude that electron diffusion in a semiconductor will only disturb the electric field lines, whereas diffusion of the magnetic field lines is successfully accounted for by the conductivity.

3 Longitudinal and Solenoidal Fields

With the foregoing discussion over, we now analyze diffusion in the time-harmonic regime. This is of importance for semiconductors but of little use for other diffusion processes, which generally occur as transients. In view of the above mentioned considerations and using the $\exp[i\omega t]$ harmonic time dependence, diffusion can be accommodated into electromagnetic field theory by means of the constitutive relations

$$\mathbf{D} = \varepsilon[\mathbf{E} + \alpha^{-2} \nabla\nabla \cdot \mathbf{E}], \qquad \mathbf{B} = \mu[\mathbf{H} + \beta^{-2} \nabla\nabla \cdot \mathbf{H}] \qquad \ldots(10a,b)$$

In these constitutive equations, the permittivity $\varepsilon = \varepsilon_s + \sigma/i\omega$, while $\alpha^2 = -i\omega\varepsilon/\varepsilon_s D_n$ with α carrying the unit of inverse-length; the permeability $\mu = \mu_s$, while the magnetic analog β of α has been introduced in Eq. (10b) for the sole purpose of generating a symmetric set of equations. In the sequel, the Faraday and Ampere-Maxwell equations

$$\nabla \times \mathbf{H} = i\omega \mathbf{D} + \mathbf{J}; \qquad \nabla \times \mathbf{E} = -i\omega \mathbf{B} - \mathbf{K} \qquad \ldots(11a,b)$$

will be used, with **J** denoting the impressed electric source, and **K** its magnetic analog.

By repeated use of Eqs (10) and (11), it is possible to obtain the governing differential equations for the **E** and **H** fields in a source-free region as

$$\nabla \times \nabla \times \mathbf{E} - k^2\alpha^{-2}\nabla\nabla . \mathbf{E} - k^2\mathbf{E} = 0 \qquad \ldots(12a)$$

$$\nabla \times \nabla \times \mathbf{H} - k^2\beta^{-2}\nabla\nabla . \mathbf{H} - k^2\mathbf{H} = 0 \qquad \ldots(12b)$$

in which $k = \omega\sqrt{(\varepsilon\mu)}$. It should be noted that the electric (resp. magnetic) diffusion constant does not enter into the magnetic (resp. electric) field equation. Furthermore, either of these equations is similar to the equation for particle displacement in elastodynamics[10].

It is well known that the elastodynamic field in a solid has two components, one of which is purely longitudinal and the other one is purely solenoidal; the two components travel with different phase velocities. In view of this analogy, let **E** and **H** be decomposed into longitudinal and solenoidal components as

$$\mathbf{E} = \mathbf{E}_s + \nabla V_e; \qquad \mathbf{H} = \mathbf{H}_s + \nabla V_m, \qquad \ldots(13a,b)$$

in which it is understood that $\nabla . \mathbf{E}_s = 0$ and $\nabla . \mathbf{H}_s = 0$. Substitution of Eq. (13a, b) into Eqs (12a) and (12b) results in four Helmholtz equations:

$$[\nabla^2 + k^2]\mathbf{E}_s = 0 \qquad \ldots(14a)$$

$$[\nabla^2 + k^2]\mathbf{H}_s = 0 \qquad \ldots(14b)$$

$$[\nabla^2 + \alpha^2]V_e = 0 \qquad \ldots(14c)$$

$$[\nabla^2 + \beta^2]V_m = 0 \qquad \ldots(14d)$$

Thus, not only is k a wavenumber, but α and β are as well. The relationships of these field components require the derivation of infinite-medium Green's functions, which will be done using the Levine-Schwinger technique.

4 Infinite-Medium Dyadic Green's Functions

The source-incorporated Helmholtz equations for the given medium can be derived from Eqs (10) and (11) as

$$\nabla \times \nabla \times \mathbf{E} - k^2\alpha^{-2}\nabla\nabla . \mathbf{E} - k^2\mathbf{E} = \mathrm{i}\omega\mu\mathbf{J} - \nabla \times \mathbf{K} \qquad \ldots(15a)$$

$$\nabla \times \nabla \times \mathbf{H} - k^2\beta^{-2}\nabla\nabla . \mathbf{H} - k^2\mathbf{H} = \mathrm{i}\omega\varepsilon\mathbf{K} + \nabla \times \mathbf{J} \qquad \ldots(15b)$$

Since the medium is linear, the solution of Eqs (15a) and (15b) can be expressed in terms of the infinite-medium Green's functions as

$$\mathbf{E}(\mathbf{r}) = \mathrm{i}\omega\mu \int d^3\mathbf{r}'\mathscr{G}_1(\mathbf{r}, \mathbf{r}') . \mathbf{J}(\mathbf{r}') + \int d^3\mathbf{r}'\mathscr{G}_2(\mathbf{r}, \mathbf{r}') . \mathbf{K}(\mathbf{r}') \qquad \ldots(16a)$$

$$\mathbf{H}(\mathbf{r}) = \int d^3\mathbf{r}'\mathscr{G}_3(\mathbf{r}, \mathbf{r}') . \mathbf{J}(\mathbf{r}') + \mathrm{i}\omega\varepsilon \int d^3\mathbf{r}'\mathscr{G}_4(\mathbf{r}, \mathbf{r}') . \mathbf{K}(\mathbf{r}') \qquad \ldots(16b)$$

in which the integrals span the source-carrying volumes, $\mathbf{r}'$ is the source point, and $\mathbf{r}$ is the field point. The aim of the Levine-Schwinger technique is to find the dyadic Green's functions $\mathscr{G}_1(\mathbf{r}, \mathbf{r}')$, etc. by considering the canonical sources $\mathbf{J} = \mathbf{J}_0\delta(\mathbf{r} - \mathbf{r}')$ and $\mathbf{K} = \mathbf{K}_0\delta(\mathbf{r} - \mathbf{r}')$, where $\delta(\mathbf{r} - \mathbf{r}')$ is the Dirac delta function.

To begin with it is necessary to dispose off the divergence terms in Eqs (10a, b), (15a) and (15b). For that purpose, the divergences of both sides of Eqs (11a, b) are taken and Eqs (10a, b) substituted into the resulting expressions. As a result, one obtains

$$[\nabla^2 + \alpha^2]\zeta_e = (\alpha^2/\mathrm{i}\omega\varepsilon)\nabla \cdot \mathbf{J};$$

$$[\nabla^2 + \beta^2]\zeta_m = (\beta^2/\mathrm{i}\omega\mu)\nabla \cdot \mathbf{K} \qquad \ldots(17a, b)$$

where

$$\zeta_e = \nabla . \mathbf{E}; \qquad \zeta_m = \nabla . \mathbf{H} \qquad \ldots(18a, b)$$

Since $\mathbf{J} = \mathbf{J}_0\ \delta(\mathbf{r} - \mathbf{r}')$ and $\mathbf{K} = \mathbf{K}_0\ \delta(\mathbf{r} - \mathbf{r}')$, the well-known solutions of Eqs (17a, b) are obtained as

$$\zeta_e(\mathbf{r}) = -(\alpha^2/\mathrm{i}\omega\varepsilon)\nabla . [g(\alpha, \mathbf{r} - \mathbf{r}')\mathbf{J}_0];$$

$$\zeta_m(\mathbf{r}) = -(\beta^2/\mathrm{i}\omega\mu)\nabla . [g(\beta, \mathbf{r} - \mathbf{r}')\mathbf{K}_0] \qquad \ldots(18c, d)$$

in which

$$g(\kappa, \mathbf{R}) = \exp[-\mathrm{i}\kappa R]/4\pi R \qquad \ldots(19)$$

is a scalar Green's function.

Using Eqs (17a, b) and the subsequent developments, it becomes possible now to obtain the Helmholtz equations for the **E** and **H** fields quite simply as

$$[\nabla^2 + k^2]\mathbf{E} = -\mathrm{i}\omega\mu\mathbf{J} + [1 - k^2\alpha^{-2}]\nabla\zeta_e + \nabla \times \mathbf{K} \qquad \ldots(20a)$$

$$[\nabla^2 + k^2]\mathbf{H} = -\mathrm{i}\omega\varepsilon\mathbf{K} + [1 - k^2\beta^{-2}]\nabla\zeta_m - \nabla \times \mathbf{J} \qquad \ldots(20b)$$

in which $[1 - k^2\alpha^{-2}]\nabla\zeta_e$ and $[1 - k^2\beta^{-2}]\nabla\zeta_m$ must be interpreted as source terms, these sources being distributed throughout the whole space as per Eq. (17). At the same time, it is to be remembered that $\mathbf{J} = \mathbf{J}_0\delta(\mathbf{r} - \mathbf{r}')$ and $\mathbf{K} = \mathbf{K}_0\delta(\mathbf{r} - \mathbf{r}')$. But as is well-known[11]

$$[\nabla \times \nabla \times \mathscr{I} - k^2\mathscr{I}] . [\mathscr{I} + k^{-2}\nabla\nabla]g(k, \mathbf{r} - \mathbf{r}') = \mathscr{I}\delta(\mathbf{r} - \mathbf{r}') \qquad \ldots(21)$$

where $\mathscr{I}$ is the idempotent. Therefore, the solution of Eq. (20) can be obtained in the form

$$\mathbf{E}(\mathbf{r}) = i\omega\mu\mathscr{G}_1(\mathbf{r},\mathbf{r}').\mathbf{J}_0 + \mathscr{G}_2(\mathbf{r},\mathbf{r}').\mathbf{K}_0 \quad \ldots(22a)$$

$$\mathbf{H}(\mathbf{r}) = \mathscr{G}_3(\mathbf{r},\mathbf{r}').\mathbf{J}_0 + i\omega\varepsilon\mathscr{G}_4(\mathbf{r},\mathbf{r}').\mathbf{K}_0 \quad \ldots(22b)$$

where the dyadic functions are given as

$$\mathscr{G}_1(\mathbf{r},\mathbf{r}') = [\mathscr{I} + k^{-2}\nabla\nabla]g(k,\mathbf{r}-\mathbf{r}') - k^{-2}\nabla\nabla g(\alpha,\mathbf{r}-\mathbf{r}') \quad \ldots(23)$$

$$\mathscr{G}_4(\mathbf{r},\mathbf{r}') = [\mathscr{I} + k^{-2}\nabla\nabla]g(k,\mathbf{r}-\mathbf{r}') - k^{-2}\nabla\nabla g(\beta,\mathbf{r}-\mathbf{r}') \quad \ldots(24)$$

$$\mathscr{G}_2(\mathbf{r},\mathbf{r}') = -\nabla\times[\mathscr{I}g(k,\mathbf{r}-\mathbf{r}')] = -\mathscr{G}_3(\mathbf{r},\mathbf{r}') \quad \ldots(25)$$

In deriving these equations, use has been made of the identity

$$\int_{\text{all space}} d^3\mathbf{r}'' g(k,\mathbf{r}-\mathbf{r}'')g(\kappa,\mathbf{r}''-\mathbf{r}')$$

$$= [g(k,\mathbf{r}-\mathbf{r}') - g(\kappa,\mathbf{r}-\mathbf{r}')]/(k^2-\kappa^2) \quad \ldots(26)$$

Coordinate-free forms of the Green's functions derived above can be obtained by explicit algebraic procedures, i.e. by using the relations[11,12]

$$\nabla\nabla g(\kappa,\mathbf{R}) = [(i\kappa R^{-1} - R^{-2})(\mathscr{I} - 3\mathbf{RR}R^{-2}) - \kappa^2\mathbf{RR}R^{-2}]g(\kappa,\mathbf{R}) \quad \ldots(27a)$$

$$\nabla\times[\mathscr{I}g(\kappa,\mathbf{R})] = (i\kappa - R^{-1})(R^{-1}\mathbf{R}\times\mathscr{I})g(\kappa,\mathbf{R}) \quad \ldots(27b)$$

It can also be shown that while $\mathscr{G}_3(\mathbf{r},\mathbf{r}')$ satisfies the differential equation

$$[\nabla\times\nabla\times\mathscr{I} - k^2\alpha^{-2}\nabla\nabla - k^2\mathscr{I}].\mathscr{G}_3(\mathbf{r},\mathbf{r}')$$

$$= \nabla\times\mathscr{I}\,\delta(\mathbf{r}-\mathbf{r}') \quad \ldots(28a)$$

$\mathscr{G}_1(\mathbf{r},\mathbf{r}')$ is a solution of

$$[\nabla\times\nabla\times\mathscr{I} - k^2\alpha^{-2}\nabla\nabla - k^2\mathscr{I}].\mathscr{G}_1(\mathbf{r},\mathbf{r}')$$

$$= \mathscr{I}\,\delta(\mathbf{r}-\mathbf{r}') \quad \ldots(28b)$$

with similar equations for $\mathscr{G}_2(\mathbf{r},\mathbf{r}')$ and $\mathscr{G}_4(\mathbf{r},\mathbf{r}')$.

5 Huygens's Principle and the Cauchy Data

Since the Green's functions are now known, it also becomes possible to derive Huygens's principle as well as explore the appropriate Cauchy data for boundary value problems. Use is made of the integral relationship

$$\int_V dv(\mathbf{b}\,.\,\text{curl curl}\,\mathbf{E} - \mathbf{E}\,.\,\text{curl curl}\,\mathbf{b})$$

$$= \int_S d\mathbf{s}\,.\,(\mathbf{E}\times\text{curl}\,\mathbf{b} - \mathbf{b}\times\text{curl}\,\mathbf{E}) \quad \ldots(29)$$

with $\mathbf{b} = i\omega\mu\mathscr{G}_1(\mathbf{r},\mathbf{r}').\mathbf{e}_p$ as the electric field due to an electric dipole of orientation $\mathbf{e}_p$ located at $\mathbf{r}' \in V$, and V is a volume enclosed by the closed surface S. $\mathbf{E}$ and the associated $\mathbf{H}$ do not have their sources in V, and the unit normal $\mathbf{e}_n = d\mathbf{s}/|d\mathbf{s}|$ points outwards from V. Use of Eqs (12a) and (28b) in Eq. (29) as well as of standard vector identities, after some algebraic tedium leads to

$$-\mathbf{E}(\mathbf{r}') = (1 - k^2\alpha^{-2})\int_S ds\,\mathbf{e}_n(\mathbf{r})\times[\mathbf{E}(\mathbf{r}).\text{curl}\,\mathscr{G}_1(\mathbf{r},\mathbf{r}') + \{\text{curl}\,\mathbf{E}(\mathbf{r})\}.\mathscr{G}_1(\mathbf{r},\mathbf{r}')] + k^2\alpha^{-2}\int_S ds\mathbf{e}_n(\mathbf{r})\cdot[\{\mathbf{E}(\mathbf{r}).\nabla\}\mathscr{G}_1(\mathbf{r},\mathbf{r}') - \{\nabla.\mathbf{E}(\mathbf{r})\}\mathscr{G}_1(\mathbf{r},\mathbf{r}')] \quad \ldots(30)$$

which reduces to the usual form[11] when $\varepsilon\alpha^{-2} = 0$. Eq. (30) represents the mathematical statement of Huygens's principle. It suggests that the field in V is completely specified by the tangential components of $\mathbf{E}$ and curl $\mathbf{E}$ on S, as well as by the normal component of $\mathbf{E}$ and the volumetric divergence of $\mathbf{E}$ on S. When $\mu\beta^{-2} \neq 0$, it should be noted that $\mathbf{H}$ cannot be directly obtained from $\mathbf{E}$ because of the diffusion term $\nabla\nabla\,.\,\mathbf{H}$. In order to obtain Huygens's principle for $\mathbf{H}$, the foregoing process is repeated to obtain the dual of Eq. (30) as

$$-\mathbf{H}(\mathbf{r}') = (1 - k^2\beta^{-2})\int_S ds\mathbf{e}_n(\mathbf{r})\times[\mathbf{H}(\mathbf{r}).\text{curl}\,\mathscr{G}_4(\mathbf{r},\mathbf{r}') + \{\text{curl}\,\mathbf{H}(\mathbf{r}).\mathscr{G}_4(\mathbf{r},\mathbf{r}')] + k^2\beta^{-2}\int_S ds\mathbf{e}_n(\mathbf{r}).[\{\mathbf{H}(\mathbf{r}).\nabla\}\mathscr{G}_4(\mathbf{r},\mathbf{r}') - \{\nabla\,.\,\mathbf{H}(\mathbf{r})\}\mathscr{G}_4(\mathbf{r},\mathbf{r}')] \quad \ldots(31)$$

Again, the Cauchy data for Eq. (31) consists of four pieces: the tangential components of $\mathbf{H}$ and curl $\mathbf{H}$ on S, as well as by the normal component of $\mathbf{H}$ and the volumetric divergence of $\mathbf{H}$ on S.

But the specification of the tangential component of curl $\mathbf{E}$ (resp. curl $\mathbf{H}$) on S is equivalent to the specification of the tangential component of $\mathbf{H}$ (resp. $\mathbf{E}$) on S, plus its volumetric divergence on S. We observe, therefore, that the electromagnetic field inside V is completely known if the tangential components, the normal components, and the volumetric divergences of both $\mathbf{E}$ and $\mathbf{H}$ are specified on S. This actually results in an overspecification, since it has been found by Poynting vector type manipulations[5] that unique results will be obtained if the tangential components of $\mathbf{E}$ and $\mathbf{H}$, and either their normal components or their divergences but not both, are specified on S.

To appreciate this intrinsic difficulty in Eqs (30) and (31), we invoke the following argument: Since $\nabla \cdot \mathbf{E}$ satisfies Eq. (17a) in the absence of sources in V, via Green's theorem it may be possible to find an integral equation relating $\nabla \cdot \mathbf{E}$ and $\mathbf{e}_n \cdot \nabla(\nabla \cdot \mathbf{E})$ on S. On the other hand, from Eq. (12a) we have

$$(\mathbf{e}_n \cdot \nabla \times)(\nabla \times \mathbf{E}) - k^2\alpha^{-2}\mathbf{e}_n \cdot \nabla(\nabla \cdot \mathbf{E}) - k^2\mathbf{e}_n \cdot \mathbf{E} = 0 \qquad \ldots(32)$$

in V. This equation relates the normal component of $\mathbf{E}$ with $\mathbf{e}_n \cdot \nabla(\nabla \cdot \mathbf{E})$ and the transversal derivatives of the tangential components of $\nabla \times \mathbf{E}$. In view of these considerations, and making use of Maxwell's equations, the Cauchy data consists of (i) $\mathbf{e}_n \cdot \mathbf{E}$, (ii) $\mathbf{e}_n \cdot \mathbf{H}$, (iii) $\mathbf{e}_n \times \mathbf{E}$, and (iv) $\mathbf{e}_n \times \mathbf{H}$ specified on S. Incidently, by letting the field point $\mathbf{r}'$ approach S normally, coupled surface integral equations can be obtained for use in the solution of scattering and radiation problems involving homogeneously doped and arbitrarily shaped semiconducting bodies.

6 Conservation of Energy

From the preceding section it is clear that in the medium concerned three types of waves can exist. The first one is a truly electromagnetic wave with a phase velocity ω/k. The second is a purely electric wave which has a phase velocity ω/α and is irrotational. So is the third one, a purely magnetic wave with a phase velocity ω/β. Thus, the mathematical correspondence with the elastodynamic case holds good, although the physics of elastic wave propagation differs from that of the electromagnetic wave propagation.

At this juncture, with the essential theoretical analysis being over, we begin to ignore the magnetic analogs β and $\mathbf{K}$ of the electric quantities α and $\mathbf{J}$; and finally, we come to the effect of diffusion on the radiated power density. One may assume that the energy flow is still $\mathbf{E} \times \mathbf{H}^*$; however, that needs to be verified. With the help of Eqs (10) and (11), it can be shown that

$$0 = \nabla \cdot (\mathbf{E} \times \mathbf{H}^*) + \mathbf{E} \cdot \mathbf{J}^* + \mathrm{i}\omega[\mu \mathbf{H} \cdot \mathbf{H}^* - \varepsilon^* \mathbf{E} \cdot \mathbf{E}^*] - \mathrm{i}\omega(\varepsilon\alpha^{-2})^* \mathbf{E} \cdot \nabla(\nabla \cdot \mathbf{E}^*) \qquad \ldots(33)$$

Let

$$\rho_d = \varepsilon_s \nabla \cdot \mathbf{E} \qquad \ldots(34a)$$

be the diffusion charge density, and a diffusion current density

$$\mathbf{J}_d = -\varepsilon_s D_n \nabla(\nabla \cdot \mathbf{E}) \qquad \ldots(34b)$$

be associated with it as per Fick's law given by Eq. (2). Then, Eq. (33) can be rewritten as

$$0 = \nabla \cdot (\mathbf{E} \times \mathbf{H}^*) + \mathbf{E} \cdot [\mathbf{J}^* + \mathbf{J}^*_d] + \mathrm{i}\omega[\mu \mathbf{H} \cdot \mathbf{H}^* - \varepsilon^* \mathbf{E} \cdot \mathbf{E}^*] \qquad \ldots(35)$$

and this equation for the conservation of energy is explainable in standard terms since diffusion has been accounted for in a manner akin to ohmic heat loss. Thus, any Poynting vector calculation proceeds as usual, but results in expressions coupling the diffusive and the wave characters of the fields.

7 Screening Potential and Induced Charges

The effect of diffusion is to modify the short range field, and not significantly the radiation zone field. From Eqs (14c) and (18a,c), it becomes quite clear that an electron cloud surrounds the electric dipole, the consequent charge distribution being given by $\rho_d(\mathbf{r}) = \varepsilon_s \nabla \cdot \mathbf{E}(\mathbf{r})$ in the far zone, as per Eq. (34a). Fick's law, given by Eq. (2), then suggests the creation of an associated current density distribution $\mathbf{J}_d(\mathbf{r}) = -\varepsilon_s D_n \nabla[\nabla \cdot \mathbf{E}(\mathbf{r})]$. Therefore, we deduce that diffusion causes the source electrostatic singularity to disappear, and screens the Coulomb field of the source up to a distance on the order of $1/\alpha''$ (where $\alpha = \alpha' - \mathrm{i}\alpha''$): If a test (negative) charge were to appear somewhere, then electrons would diffuse to screen out its Coulomb field. We conclude therefore that diffusion will play a very important role where the geometry is characterised by lengths comparable to or smaller than $1/\alpha''$, which length may be considered as the dividing line between the diffusion-dominated and the wave-dominated length scales.

The effect of diffusion is to create potential energy barriers for the propagation of the electromagnetic field; in other words, the radiation resistance of the medium is enhanced by the presence of α. Additionally, radiated electric charges due to one source are also scattered by the diffusion barriers created by another source, thereby further complicating interactions between sources.

To be sure, $\mathbf{J}_d$ and ρ_d of Eq. (34a, b) satisfy Fick's law [Eq. (2)]; in addition, Eq. (11a) can be written in the form

$$\nabla \times \mathbf{H} = \mathrm{i}\omega\varepsilon\mathbf{E} + \mathbf{J}_d + \mathbf{J} \qquad \ldots(36)$$

so as to account for diffusion explicitly. But $\mathbf{J}_d$ and ρ_d do not satisfy Eq. (1) for conservation of charge. If we, however, suggest the introduction of additional charges and currents via

$$\mathbf{J}_c = -\sigma\mathbf{E}, \qquad \rho_c = (\sigma/\mathrm{i}\omega)\nabla \cdot \mathbf{E} \qquad \ldots(37a, b)$$

which do satisfy Eq. (1) by themselves, then it is easy to see using Eq. (17a), that

$$\nabla \cdot \mathbf{J}_d + \mathrm{i}\omega(\rho_d + \rho_c) = 0 \qquad \ldots(38)$$

This turns out to be sensible, since the same electrons occur in both the conduction and the diffusion mechanisms: a conduction electron may 'disappear' and become a diffusion electron, and vice versa. As expected, when $D_n \to 0$, then $\mathbf{J}_d$, ρ_d and ρ_c all vanish outside the source region, which is where they are pertinent; electron diffusion is not a stand-alone process and must be accompanied by conduction.

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