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FRACTALS AND *THE CAT IN THE HAT*

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ABSTRACT

This is in the nature of a confession:
Dr. Seuss helped me comprehend fractals.

Sometime in July 1984, me colleague Russell Messier came in for a chat and ended up introducing me to *fractals* [1]. Being a materials scientist, he, of course, tried to explain fractals in terms of the growth of thin films. But, not being a materials scientist, I had considerable difficulty in visualizing fractals in SEMs and TEMs of pyrolytic graphite films. Particularly, I also did not, and still am not able to, distinguish between ballistic aggregation and diffusion-limited aggregation. I did understand the triadic Koch snowflake, but that had been introduced to me by Prof. V. Menon of the Banaras Hindu University, who had taught me operations research in 1978 when I was an undergraduate at that university.

Fractals were sufficiently intriguing, nevertheless; the beautiful computer-generated images of the Mandelbrot set were too intricate to leave me unaffected. I ordered a copy of Mandelbrot's *The Fractal Geometry of Nature* [1], marveled at the colorful pictures, and tried to read that book. I do not mind acknowledging here, with apologies to Prof. Mandelbrot, that I found it very taxing to read his book; in fact, I have not yet finished reading it. Whereas the Koch snowflake, the Sierpinski gasket, and the Menger sponge could be understood by me somewhat, I could make no sense of such concepts as *lacunarity* and *intermittency.* And I spent almost a whole year on this activity!

Enlightenment came in September 1985. One day, it came to me that I had frequently read Dr. Seuss to a young son of a friend of mine several years earlier. Since I had recently become the proud father of a baby girl, I decided to pay a visit to a children's bookstore and reacquaint myself with Dr. Seuss and his hilarious belly-contorting creations. Imagine my surprise when I leafed through *The Cat in the Hat Comes Back* [2] at the bookstore, and the mystery went out of fractals in a trice.

The object of this brief essay is to report on the connection between fractals and the CAT. I will begin by recapitulating the story. A brother and sister have been left alone in the house, while their parents are away on some business. The CAT comes in, and, despite the childrens' protests, naughtily settles down in the bath tub. When he gets out, there is

A big long pink cat ring!
It looked like pink ink!

left in the tub. This must be removed before Mom comes back, and the CAT does put in a heroic, but unsatisfactory, effort in which the pink matter is transferred successively from one object to another.

Eventually, the CAT hits upon a brilliant stratagem: he releases a smaller cat A who resides in his hat. For further assistance, cat A summons cat B who stays in cat A's hat. Cat B seeks assistance from cat C, and so on, until finally there came cat Z who is the *habitué* of cat Y's hat. But, cat

Z is too small to see.
So don't try. You cannot.

By this time, the pink pigment, which had been removed from the bath tub, has instead spread on the snow outside, and all around there is pink snow. Cat Z, however, has something (super snow cleaner?) called VOOM, which is applied to the problem at hand. Not only does the pink snow become white again, but all the cats from cat A to cat Z also get back into the hats of their respective hierarchical superiors as soon as VOOM is applied. The CAT thereupon bids the children goodbye just about as Mom returns.

The hierarchical nature of the twenty-seven cats in this tale is very apparent. Assuming that cat A and its hat are the exact replicas, respectively, of the CAT and ITS HAT, and so on, there must be scaling laws for the sizes of the cats. One could in that case deduce a *similarity dimension* for the cats, say as the ratio

$$\frac{\log(\text{volume of cat M's tail} \div \text{volume of cat N's tail})}{\log(\text{area of cat M's paw} \div \text{area of cat N's paw})}$$

Of course, several such similarity dimensions based on different anatomical of the cats could be found! Would they all agree, necessarily? That leads to the question of self-similarity *versus* self-affinity [3], or, in other words, to the recently-emerged concept of *multifractality* [4].

Cat Z is very interesting: it is too small to see. The CAT, on the other hand, is very much observable but there is definitely an upper limit on its size. This brings us to distinguish between strictly *mathematical* fractals—such as the Menger sponge—and the more commonly observed *natural* fractals. In nature, it would appear that there is a lower bound on length. Certainly, an experimenter is limited by the smallest object his/her probe can resolve. On the other hand, though the size of the universe may be an imponderable, one can safely state that no experimenter can measure an object larger than a certain one in a reasonable period of time. Thus, natural fractals are possibly *band-limited* in size [4], at least insofar as we can perceive: but there are no corresponding limitations on the magnification/demagnification of the Menger sponge, and one can continue on *ad nauseam.*

One can also think of the twenty-seven cats as the members of a *time-series* which folds into a *strange attractor* [5]. In this case, cat Z would be the strange attractor, for no other cat followed it.

Let us now finally look at the pink material. At its largest, it covered a certain amont of terrain around the house. But it did not grow any larger in its coverage after cat Y had arrived on the scene. The analogy of the pink snowy terrain with the *basin* [1] of a strange attractor appears to be very attractive. Particularly more so, when one thinks of VOOM: now the strange attractor, of course, is the location of the hat of the CAT at the instant VOOM was applied.

For the record, I will note here that some of the foregoing was presented by me at a conference on electromagnetic fields at Philadelphia [6].

I will take my leave with the following thought: there are fractals galore waiting to be discovered in the chaotic works of Dr. Seuss. Enterprising readers may devote some time and energy to discovering these *juvenile* fractals; in time to come, the books of Dr. Seuss, probably much to his own surprise, may turn out to be pedagogical tools for training future generations of fractal researchers.[1]

References:

1. B. B. Mandelbrot, *The Fractal Geometry of Nature,* Freeman, Boston, Massachusetts, 1983.
2. Dr. Seuss, *The Cat in the Hat Comes Back,* Beginner Books, New York, 1958.
3. A. Lakhtakia, R. Messier, V. V. Varadan, and V. K. Varadan, Self-Similarity versus Self-Affinity: The Sierpinski Gasket Revisited, *Journal of Physics, A*:19, pp. L985-989, 1986.
4. T. Vicsek, *Fractal Growth Phenomena,* World Scientific, Singapore, 1989.
5. F. C. Moon, *Chaotic Vibrations,* Wiley, New York, 1987.
6. A. Lakhtakia, V. K. Varadan, and V. V. Varadan, Self-Similar Fractals and Fourier Transforms, *US National Radio Science Meeting,* Philadelphia, Pennsylvania, June 6-13, 1986.

[1] I mean researchers investigating fractals, not researchers who are fractal!

Further Suggested Reading:

Gleick, J., *Chaos: Making a New Science,* Penguin Books, New York, 1988.

Note Added in Proof: Christmas 1989 gave me an unexpected present. I saw the Dr. Seuss movie "Horton Hears a Who."

ATTENTION

The number ten is imbedded in many English words, as in tent. Ten others are defined below. What are they? If your attention wavers, reduce the tension by looking on page 175.

1. Frequently
2. Sinew
3. To fasten the hatches
4. A forewarning
5. Taut
6. Offensive smell
7. Observant
8. Extremely loud
9. Rendering dark or gloomy
10. Cutting tool

Charles W. Trigg