J. Phys. France 51 (1990) 2235–2242 15 OCTOBRE 1990, PAGE 2235

Classification
Physics Abstracts
41.10

Short Communication

Polarizability dyadics of small bianisotropic spheres

Akhlesh Lakhtakia

Department of Engineering Science and Mechanics, The Pennsylvania State University, University Park, PA 16802, U.S.A.

(Received 23 April 1990, accepted 6 August 1990)

Abstract. — The low-frequency scattering response of a homogeneous sphere made of a general linear bianisotropic material is quantified in terms of polarizability dyadics. The polarizability dyadics are then used to generalize the Maxwell-Garnet model for a composite obtained by randomly suspending small bianisotropic spheres in an isotropic achiral host medium.

1. Introduction.

Treatment of scattering by homogeneous isotropic spheres can be traced back to Mie [1] and Debye [2] ; the resulting formulation has enjoyed enormous popularity, and has been extended to bi-isotropic spheres as well [3] . But a treatment of comparable simplicity, elegance and generality for homogeneous anisotropic spheres is still elusive. Analytic solutions requiring intensive computation have become available for scattering by radially uniaxial dielectric spheres [4, 5] . In recent years, a semi-microscopic numerical approach due to Purcell and Pennypacker [6] has been used for anisotropic spheres [7] . For electrically small anisotropic spheres, however, the most common procedure used is to compute the scattering characteristics of an "averaged" isotropic sphere [7-9].

It is well-known that the field scattered by any scatterer (in free space) can be decomposed into multipoles [10] . In particular, at low enough frequencies, the scattering response of a homogeneous dielectric sphere is isomorphic with the radiation characteristics of a point electric dipole; hence, a small dielectric sphere may be adequately characterized by an electric polarizability [9] . This fact, along with the Clausius-Mosotti relation [10] , was utilized by Maxwell-Garnet [11] to fashion a theory for the macroscopic properties of a composite medium constructed by randomly dispersing small spherical inclusions in a dielectric host material. The resulting approach has been considerably augmented for different cases [12, 13] , and it has its competitors in the Bruggeman model [14] and its variants [e.g., 15], as well as in the more rigorous multiple scattering theories [16-18]. The elegant simplicity of this approach has lead to its extensive usage, nevertheless, despite its many limitations [12, 13] .

The objectives of this paper are twofold. First, the scattering response of a general, homogeneous, small bianisotropic [19, 20] sphere will be obtained. Second, the Maxwell-Garnet model will be generalized in order to estimate the effective properties of a composite constructed by randomly suspending small bianisotropic spheres in a homogeneous, isotropic, achiral [21] host medium; without loss of generality, the host medium will be taken to be free space. Coupled volume integral equations will be used in section 2 to obtain the polarizability dyadics of a small bianisotropic sphere suspended in free space. Parenthetically, it should be noted that these polarizability dyadics can be used in extending the Purcell-Pennypacker technique [6] for arbitrary scatterers [e.g., 7, 22]. Following the developments of section 2, the constitutive parameters of the Maxwell-Garnet model of the composite will be estimated in section 3.

2. Polarizabilities of a small bianisotropic sphere.

Let a spherical region of radius a be occupied by a general, linear, homogeneous medium specified by the constitutive equations

$$\mathbf{D}(\mathbf{r}) = \varepsilon_0 \left[\underline{\underline{\varepsilon}} \bullet \mathbf{E}(\mathbf{r}) + \underline{\underline{\alpha}} \bullet \mathbf{H}(\mathbf{r}) \right], \qquad \mathbf{B}(\mathbf{r}) = \mu_0 \left[\underline{\underline{\beta}} \bullet \mathbf{E}(\mathbf{r}) + \underline{\underline{\mu}} \bullet \mathbf{H}(\mathbf{r}) \right], \mathbf{r} \in \mathrm{V} \tag{1}$$

be embedded in free space (ε_0, μ_0). The sphere is assumed to be centered at the origin $\mathbf{r} = \mathbf{0}$, $\underline{\underline{\varepsilon}}$ is the relative permittivity dyadic, $\underline{\underline{\mu}}$ is the relative permeability dyadic, while $\underline{\underline{\alpha}}$ and $\underline{\underline{\beta}}$ represent the magnetoelectric dyadics; no restrictions have been placed as of now on these dyadics, and a medium described by (1) is called bianisotropic [19, 20].

Bianisotropic media occur readily in nature. Materials exhibiting the dielectric Faraday effect $\left(\underline{\underline{\mu}} = \mu \, \underline{\underline{I}}, \ \underline{\underline{\alpha}} = \underline{\underline{\beta}} = 0\right)$ abound as crystals [23]. Ferrites and plasmas [19] exhibit the magnetic Faraday effect $\left(\underline{\underline{\varepsilon}} = \varepsilon \, \underline{\underline{I}}, \ \underline{\underline{\alpha}} = \underline{\underline{\beta}} = 0\right)$. Natural optically active materials $\left(\underline{\underline{\varepsilon}} = \varepsilon \, \underline{\underline{I}}, \ \underline{\underline{\mu}} = \mu \, \underline{\underline{I}}, \ \underline{\underline{\alpha}} = \alpha \, \underline{\underline{I}}, \ \underline{\underline{\beta}} = \beta \, \underline{\underline{I}}\right)$ are well-known to organic and physical chemists [21, 24]. Even a simply moving, isotropic dielectric scatterer appears to be bianisotropic to a stationary observer [25]. Thus, a material characterised by (1) is the most general, linear, non-diffusive electromagnetic substance. Physically realizable forms of the constitutive tensors have been discussed at length by Post [19] within the framework of Lorentz covariance [26].

In the absence of any impressed sources, Maxwell curl equations satisfied by time-harmonic $[e^{-i\omega t}]$ electromagnetic fields everywhere can be expressed concisely as

$$\nabla \times \mathbf{H}(\mathbf{r}) + i\omega\varepsilon_0 \mathbf{E}(\mathbf{r}) = \mathbf{J}(\mathbf{r}), \qquad \nabla \times \mathbf{E}(\mathbf{r}) - i\omega\mu_0 \mathbf{H}(\mathbf{r}) = -\mathbf{K}(\mathbf{r}). \tag{2}$$

The quantities on the right-hand sides of (2) are given by

$$\mathbf{J}(\mathbf{r}) = \mathbf{0}, \qquad \mathbf{r} \notin \mathrm{V}, \tag{3a}$$

$$\mathbf{J}(\mathbf{r}) = -i\omega\varepsilon_0 \left[\left(\underline{\underline{\varepsilon}} - \underline{\underline{I}} \right) \bullet \mathbf{E}(\mathbf{r}) + \underline{\underline{\alpha}} \bullet \mathbf{H}(\mathbf{r}) \right], \quad \mathbf{r} \in \mathrm{V}, \tag{3b}$$

$$\mathbf{K}(\mathbf{r}) = \mathbf{0}, \qquad \mathbf{r} \notin \mathrm{V}, \tag{4a}$$

$$\mathbf{K}(\mathbf{r}) = -i\omega\mu_0 \left[\underline{\underline{\beta}} \bullet \mathbf{E}(\mathbf{r}) + \left(\underline{\underline{\mu}} - \underline{\underline{I}} \right) \bullet \mathbf{H}(\mathbf{r}) \right], \quad \mathbf{r} \in \mathrm{V}, \tag{4b}$$

In which $\underline{\underline{I}}$ is the idempotent. Thus, the influence of the bianisotropic scatterer is being treated as that due to certain volume distributions of the electric and the magnetic current densities in free space.

The solution of (2) is well-known, and is given for all $\mathbf{r}$ by [27]

$$\mathbf{E}(\mathbf{r}) - \mathbf{E}_{\rm inc}(\mathbf{r}) = i\omega\mu_0 \int_{\rm V} {\rm d}v'\ \underline{\underline{G}}_0\,(\mathbf{r},\mathbf{r}') \bullet \mathbf{J}(\mathbf{r}') - \nabla \times \int_{\rm V} {\rm d}v'\ \underline{\underline{G}}_0\,(\mathbf{r},\mathbf{r}') \bullet \mathbf{K}(\mathbf{r}'), \tag{5a}$$

$$\mathbf{H}(\mathbf{r}) - \mathbf{H}_{\rm inc}(\mathbf{r}) = i\omega\varepsilon_0 \int_{\rm V} {\rm d}v'\ \underline{\underline{G}}_0\,(\mathbf{r},\mathbf{r}') \bullet \mathbf{K}(\mathbf{r}') + \nabla \times \int_{\rm V} {\rm d}v'\ \underline{\underline{G}}_0\,(\mathbf{r},\mathbf{r}') \bullet \mathbf{J}(\mathbf{r}'). \tag{5b}$$

In these equations, the free space Green's dyadic $\underline{\underline{G}}_0\,(\mathbf{r},\mathbf{r}')$ is defined as

$$\underline{\underline{G}}_0\,(\mathbf{r},\mathbf{r}') = \left(\underline{\underline{I}} + \nabla\nabla/k_0^2\right) g_0(\mathbf{r},\mathbf{r}'), \tag{6a}$$

where

$$g_0(\mathbf{r},\mathbf{r}') = \ \exp\ [ik_0|\mathbf{r}-\mathbf{r}'|]\,/4\pi|\mathbf{r}-\mathbf{r}'|, \tag{6b}$$

is the free space scalar Green's function, $k_0 = \omega\sqrt{(\varepsilon_0\mu_0)}$ is the free space wavenumber, and $\mathbf{E}_{\rm inc}(\mathbf{r})$ and $\mathbf{H}_{\rm inc}(\mathbf{r})$ represent the field incident on the scatterer. The specific properties of the solution (5) have been discussed in detail by Jones [27] , to which work the interested reader is referred to for further details.

It is assumed here that $k_0 a \ll 1$; hence, the field inside V can be easily estimated by making a long-wavelength approximation. First, the equations (5) can be transformed to [28]

$$\begin{aligned}\mathbf{E}(\mathbf{r}) = \mathbf{E}_{\rm inc}(\mathbf{r}) + \mathbf{J}(\mathbf{r})/\left(3i\omega\varepsilon_0\right) + {\rm P.V.}\Big\{ & i\omega\mu_0 \int_{\rm V} {\rm d}v'\ \mathbf{J}(\mathbf{r}')\bullet \underline{\underline{G}}_0\,(\mathbf{r}',\mathbf{r}) - \mathbf{J}(\mathbf{r})/\left(3i\omega\varepsilon_0\right) \\ & - \int_{\rm V} {\rm d}v'\ \mathbf{K}(\mathbf{r}') \bullet \left[\nabla' \times \underline{\underline{G}}_0\,(\mathbf{r}',\mathbf{r})\right]\Big\}\,;\mathbf{r} \in {\rm V},\end{aligned} \tag{7a}$$

$$\begin{aligned}\mathbf{H}(\mathbf{r}) = \mathbf{H}_{\rm inc}(\mathbf{r}) + \mathbf{K}(\mathbf{r})/\left(3i\omega\mu_0\right) + {\rm P.V.}\Big\{ & i\omega\varepsilon_0 \int_{\rm V} {\rm d}v'\ \mathbf{K}(\mathbf{r}')\bullet \underline{\underline{G}}_0\,(\mathbf{r}',\mathbf{r}) - \mathbf{K}(\mathbf{r})/\left(3i\omega\mu_0\right) \\ & + \int_{\rm V} {\rm d}v'\ \mathbf{J}(\mathbf{r}') \bullet \left[\nabla' \times \underline{\underline{G}}_0\,(\mathbf{r}',\mathbf{r})\right]\Big\}\,;\mathbf{r} \in {\rm V},\end{aligned} \tag{7b}$$

in which P.V. denotes the principal value, while the singularities at $\mathbf{r} = \mathbf{r}'$ have been clearly identified. Parenthetically, it is to be noted here that (7a, b) considerably generalize and extend Jones' formulation [27] . Next, the P.V.$\{\bullet\}$ are set to zero since the scatterer is electrically small, and one gets

$$\mathbf{E}(0) = \mathbf{E}_{\rm inc}(0) + \mathbf{J}(0)/\left(3i\omega\varepsilon_0\right), \quad \mathbf{H}(0) = \mathbf{H}_{\rm inc}(0) + \mathbf{K}(0)/\left(3i\omega\mu_0\right). \tag{8a, b}$$

It must be remarked here that the spheres considered here have radii small compared to the free-space wavelength at a given frequency ω. From available literature concerning isotropic dielectric spheres, it appears that the approximation (8a, b) may be valid for $k_0 a < 0.1$ or thereabouts [29, 30].

A long-wavelength approximation is also made for the scattered field in the far-zone, and leads to

$$\mathbf{E}_{\rm sc}(\mathbf{r}) \sim \left\{\omega^2\mu_0\left[\mathcal{I} - \mathbf{u}_r\mathbf{u}_r\right] \bullet \mathbf{p} - \omega k_0 \mathbf{u}_r \times \mathbf{m}\right\}\ \exp\ [ik_0 r]\,/4\pi r, \tag{9a}$$

$$\mathbf{H}_{\rm sc}(\mathbf{r}) \sim \left\{\omega^2\varepsilon_0\left[\mathcal{I} - \mathbf{u}_r\mathbf{u}_r\right] \bullet \mathbf{m} + \omega k_0 \mathbf{u}_r \times \mathbf{p}\right\}\ \exp\ [ik_0 r]\,/4\pi r. \tag{9b}$$

in which $\mathbf{u}_r$ is a unit vector parallel to $\mathbf{r}$, while

$$\mathbf{p} = (i/\omega) \int_V dv' \mathbf{J}(\mathbf{r}'), \tag{10a}$$

is an electric dipole moment, and

$$\mathbf{m} = (i/\omega) \int_V dv' \mathbf{K}(\mathbf{r}'), \tag{10b}$$

is a magnetic dipole moment. Consequently, in the low-frequency regime, the scattered fields attributable to the bianisotropic scatterer can be thought of as due to the combination of an electric dipole and a magnetic dipole.

Next, it is assumed that the electromagnetic field induced inside the small sphere is spatially constant; *i.e.*, $\mathbf{E}(\mathbf{r}) = \mathbf{E}(0)$ and $\mathbf{H}(\mathbf{r}) = \mathbf{H}(0)$ for all $\mathbf{r} \in V$. Solving (8) and (10) together yields

$$\mathbf{p} = \underline{\underline{a}}_{ee} \bullet \mathbf{E}_{inc} + \underline{\underline{a}}_{em} \bullet \mathbf{H}_{inc}, \qquad \mathbf{m} = \underline{\underline{a}}_{me} \bullet \mathbf{E}_{inc} + \underline{\underline{a}}_{mm} \bullet \mathbf{H}_{inc}, \tag{11}$$

in which the four polarizability dyadics for the small bianisotropic sphere are given by

$$\underline{\underline{a}}_{ee} = 4\pi a^3 \varepsilon_0 \left(\underline{\underline{\varepsilon}} + 2\,\underline{\underline{I}}\right)^{-1} \bullet \left[\left(\underline{\underline{\varepsilon}} - \underline{\underline{I}}\right) + 3\,\underline{\underline{\alpha}} \bullet \underline{\underline{\Delta}}_e^{-1} \bullet \underline{\underline{\alpha}}^{-1}\right], \tag{12a}$$

$$\underline{\underline{a}}_{em} = -12\pi a^3 \varepsilon_0\, \underline{\underline{\Delta}}_h^{-1} \bullet \underline{\underline{\beta}}^{-1}, \tag{12b}$$

$$\underline{\underline{a}}_{me} = -12\pi a^3 \mu_0\, \underline{\underline{\Delta}}_e^{-1} \bullet \underline{\underline{\alpha}}^{-1}, \tag{12c}$$

$$\underline{\underline{a}}_{mm} = 4\pi a^3 \mu_0 \left(\underline{\underline{\mu}} + 2\,\underline{\underline{I}}\right)^{-1} \bullet \left[\left(\underline{\underline{\mu}} - \underline{\underline{I}}\right) + 3\,\underline{\underline{\beta}} \bullet \underline{\underline{\Delta}}_h^{-1} \bullet \underline{\underline{\beta}}^{-1}\right]. \tag{12d}$$

In these equations, $\underline{\underline{\beta}}^{-1}$ is the dyadic inverse of $\underline{\underline{\beta}}$, etc.; while

$$\underline{\underline{\Delta}}_e = \underline{\underline{I}} - \underline{\underline{\alpha}}^{-1} \bullet \left(\underline{\underline{\varepsilon}} + 2\,\underline{\underline{I}}\right) \bullet \underline{\underline{\beta}}^{-1} \bullet \left(\underline{\underline{\mu}} + 2\,\underline{\underline{I}}\right), \tag{13a}$$

and

$$\underline{\underline{\Delta}}_h = \underline{\underline{I}} - \underline{\underline{\beta}}^{-1} \bullet \left(\underline{\underline{\mu}} + 2\,\underline{\underline{I}}\right) \bullet \underline{\underline{\alpha}}^{-1} \bullet \left(\underline{\underline{\varepsilon}} + 2\,\underline{\underline{I}}\right) \cdot \tag{13b}$$

It is assumed here and hereafter that all dyadic inverses exist [19 (Chaps. 6 and 8)] or can be satisfactorily taken into account.

Expressions (12a-d) constitute the most general result for the polarizability-representation of a small homogeneous sphere made of the most general, linear, non-diffusive material and embedded in free space. Since they hold for fully bianisotropic spheres, these results subsume not only the traditional result for isotropic dielectric spheres [10] but also that for chiral spheres [22, 28, 31].

3. Effective properties of a composite.

Finally, a discrete random medium made up of identical bianisotropic spheres distributed in a free space is considered. The spheres are randomly distributed with N being the number density per unit volume. It is also assumed in this section that the volumetric proportion of the chiral material is small, and it is expected that the analysis will hold for $N(4\pi/3)a^3 \leq 0.1$ or so [12]. Finally, it is also assumed here that all spheres have identical constitutive tensors within a global coordinate system.

If the resulting *composite medium* is to be viewed as being *effectively homogeneous*, its constitutive parameters must be of the form

$$\mathbf{D} = \varepsilon_0 \left[\underline{\underline{\varepsilon_{\text{eff}}}} \bullet \mathbf{E} + \underline{\underline{\alpha_{\text{eff}}}} \bullet \mathbf{H} \right], \qquad \mathbf{B} = \mu_0 \left[\underline{\underline{\beta_{\text{eff}}}} \bullet \mathbf{E} + \underline{\underline{\mu_{\text{eff}}}} \bullet \mathbf{H} \right]. \tag{14}$$

The concept of flux densities, $\mathbf{D}$ and $\mathbf{B}$, implies a polarization field $\mathbf{P} = (\mathbf{D} - \varepsilon_0 \mathbf{E})$ and a magnetization field $\mathbf{M} = (\mathbf{B} - \mu_0 \mathbf{H})$. The polarization field $\mathbf{P}$ is defined as the electric dipole moment per unit volume, while the magnetization field $\mathbf{M}$ is the magnetic dipole moment per unit volume; thus,

$$\mathbf{P} = N\mathbf{p}, \qquad \mathbf{M} = N\mathbf{m}. \tag{15}$$

The equivalent moments, $\mathbf{p}$ and $\mathbf{m}$, of a single sphere are proportional to the local (Lorentz) electric and magnetic fields exciting it [10]. Therefore, from (11),

$$\mathbf{p} = \underline{\underline{a_{\text{ee}}}} \bullet \mathbf{E}_{\text{L}} + \underline{\underline{a_{\text{em}}}} \bullet \mathbf{H}_{\text{L}}, \qquad \mathbf{m} = \underline{\underline{a_{\text{me}}}} \bullet \mathbf{E}_{\text{L}} + \underline{\underline{a_{\text{mm}}}} \bullet \mathbf{H}_{\text{L}}, \tag{16}$$

must be used in (15); here, the subscript "L" stands for Lorentz. Assuming that spheres are weakly anisotropic, the usual [13, 32] prescription for the Lorentz field can be followed; *i.e.*,

$$\mathbf{E}_{\text{L}} = \mathbf{E} + \mathbf{P}/(3\varepsilon_0), \qquad \mathbf{H}_{\text{L}} = \mathbf{H} + \mathbf{M}/(3\mu_0) \cdot \tag{17}$$

Simultaneous solution of (15)-(17) then yields

$$\begin{aligned} \mathbf{P} = &\left\{ (3\mu_0/N)\, \underline{\underline{a_{\text{em}}^{-1}}} \bullet \left[\underline{\underline{I}} - (N/3\varepsilon_0)\, \underline{\underline{a_{\text{ee}}}} \right] - \left[\underline{\underline{I}} - (N/3\mu_0)\, \underline{\underline{a_{\text{mm}}}} \right]^{-1} \bullet (N/3\varepsilon_0)\, \underline{\underline{a_{\text{me}}}} \right\}^{-1} \bullet \\ &\bullet \left\{ \left[\underline{\underline{I}} - (N/3\mu_0)\, \underline{\underline{a_{\text{mm}}}} \right]^{-1} \bullet N\, \underline{\underline{a_{\text{me}}}} + 3\mu_0\, \underline{\underline{a_{\text{em}}^{-1}}} \bullet \underline{\underline{a_{\text{ee}}}} \right\} \bullet \mathbf{E} + \\ &+ \left\{ (3\mu_0/N)\, \underline{\underline{a_{\text{em}}^{-1}}} \bullet \left[\underline{\underline{I}} - (N/3\varepsilon_0)\, \underline{\underline{a_{\text{ee}}}} \right] - \left[\underline{\underline{I}} - (N/3\mu_0)\, \underline{\underline{a_{\text{mm}}}} \right]^{-1} \bullet (N/3\varepsilon_0)\, \underline{\underline{a_{\text{me}}}} \right\}^{-1} \bullet \\ &\bullet \left\{ \left[\underline{\underline{I}} - (N/3\mu_0)\, \underline{\underline{a_{\text{mm}}}} \right]^{-1} \bullet N\, \underline{\underline{a_{\text{mm}}}} + 3\mu_0\, \underline{\underline{I}} \right\} \bullet \mathbf{H}, \end{aligned} \tag{18a}$$

$$\begin{aligned}\mathbf{M} = &\left\{(3\varepsilon_0/N)\ \underline{\underline{a}}_{\mathrm{me}}^{-1} \bullet \left[\underline{\underline{I}} - (N/3\mu_0)\ \underline{\underline{a}}_{\mathrm{mm}}\right] - \left[\underline{\underline{I}} - (N/3\varepsilon_0)\ \underline{\underline{a}}_{\mathrm{ee}}\right]^{-1} \bullet (N/3\mu_0)\ \underline{\underline{a}}_{\mathrm{em}}\right\}^{-1} \bullet \\ &\bullet \left\{\left[\underline{\underline{I}} - (N/3\varepsilon_0)\ \underline{\underline{a}}_{\mathrm{ee}}\right]^{-1} \bullet N\ \underline{\underline{a}}_{\mathrm{em}} + 3\varepsilon_0\ \underline{\underline{a}}_{\mathrm{me}}^{-1} \bullet \underline{\underline{a}}_{\mathrm{mm}}\right\} \bullet \mathbf{H} + \\ &+ \left\{(3\varepsilon_0/N)\ \underline{\underline{a}}_{\mathrm{me}}^{-1} \bullet \left[\underline{\underline{I}} - (N/3\mu_0)\ \underline{\underline{a}}_{\mathrm{mm}}\right] - \left[\underline{\underline{I}} - (N/3\varepsilon_0)\ \underline{\underline{a}}_{\mathrm{ee}}\right]^{-1} \bullet (N/3\mu_0)\ \underline{\underline{a}}_{\mathrm{em}}\right\}^{-1} \bullet \\ &\bullet \left\{\left[\underline{\underline{I}} - (N/3\varepsilon_0)\ \underline{\underline{a}}_{\mathrm{ee}}\right]^{-1} \bullet N\ \underline{\underline{a}}_{\mathrm{ee}} + 3\varepsilon_0\ \underline{\underline{I}}\right\} \bullet \mathbf{E}.\end{aligned} \tag{18b}$$

It follows from (14), as well as from the definitions of **P** and **M**, that the constitutive parameters of the effective medium can *now* be estimated as

$$\begin{aligned}\underline{\underline{\varepsilon}}_{\mathrm{eff}} - \underline{\underline{I}} = &\left\{(3\mu_0/N)\ \underline{\underline{a}}_{\mathrm{em}}^{-1} \bullet \left[\underline{\underline{I}} - (N/3\varepsilon_0)\ \underline{\underline{a}}_{\mathrm{ee}}\right] - \left[\underline{\underline{I}} - (N/3\mu_0)\ \underline{\underline{a}}_{\mathrm{mm}}\right]^{-1} \bullet (N/3\varepsilon_0)\ \underline{\underline{a}}_{\mathrm{me}}\right\}^{-1} \bullet \\ &\bullet (1/\varepsilon_0) \left\{\left[\underline{\underline{I}} - (N/3\mu_0)\ \underline{\underline{a}}_{\mathrm{mm}}\right]^{-1} \bullet N\ \underline{\underline{a}}_{\mathrm{me}} + 3\mu_0\ \underline{\underline{a}}_{\mathrm{em}}^{-1} \bullet \underline{\underline{a}}_{\mathrm{ee}}\right\},\end{aligned} \tag{19a}$$

$$\begin{aligned}\underline{\underline{\alpha}}_{\mathrm{eff}} = &\left\{(3\mu_0/N)\ \underline{\underline{a}}_{\mathrm{em}}^{-1} \bullet \left[\underline{\underline{I}} - (N/3\varepsilon_0)\ \underline{\underline{a}}_{\mathrm{ee}}\right] - \left[\underline{\underline{I}} - (N/3\mu_0)\ \underline{\underline{a}}_{\mathrm{mm}}\right]^{-1} \bullet (N/3\varepsilon_0)\ \underline{\underline{a}}_{\mathrm{me}}\right\}^{-1} \bullet \\ &\bullet (1/\varepsilon_0) \left\{\left[\underline{\underline{I}} - (N/3\mu_0)\ \underline{\underline{a}}_{\mathrm{mm}}\right]^{-1} \bullet N\ \underline{\underline{a}}_{\mathrm{mm}} + 3\mu_0\ \underline{\underline{\mathrm{I}}}\right\},\end{aligned} \tag{19b}$$

$$\begin{aligned}\underline{\underline{\mu}}_{\mathrm{eff}} - \underline{\underline{I}} = &\left\{(3\varepsilon_0/N)\ \underline{\underline{a}}_{\mathrm{me}}^{-1} \bullet \left[\underline{\underline{I}} - (N/3\mu_0)\ \underline{\underline{a}}_{\mathrm{mm}}\right] - \left[\underline{\underline{I}} - (N/3\varepsilon_0)\ \underline{\underline{a}}_{\mathrm{ee}}\right]^{-1} \bullet (N/3\mu_0)\ \underline{\underline{a}}_{\mathrm{em}}\right\}^{-1} \bullet \\ &\bullet (1/\mu_0) \left\{\left[\underline{\underline{I}} - (N/3\varepsilon_0)\ \underline{\underline{a}}_{\mathrm{ee}}\right]^{-1} \bullet N\ \underline{\underline{a}}_{\mathrm{em}} + 3\varepsilon_0\ \underline{\underline{a}}_{\mathrm{me}}^{-1} \bullet \underline{\underline{a}}_{\mathrm{mm}}\right\},\end{aligned} \tag{19c}$$

$$\begin{aligned}\underline{\underline{\beta}}_{\mathrm{eff}} = &\left\{(3\varepsilon_0/N)\ \underline{\underline{a}}_{\mathrm{me}}^{-1} \bullet \left[\underline{\underline{I}} - (N/3\mu_0)\ \underline{\underline{a}}_{\mathrm{mm}}\right] - \left[\underline{\underline{I}} - (N/3\varepsilon_0)\ \underline{\underline{a}}_{\mathrm{ee}}\right]^{-1} \bullet (N/3\mu_0)\ \underline{\underline{a}}_{\mathrm{em}}\right\}^{-1} \bullet \\ &\bullet (1/\mu_0) \left\{\left[\underline{\underline{I}} - (N/3\varepsilon_0)\ \underline{\underline{a}}_{\mathrm{ee}}\right]^{-1} \bullet N\ \underline{\underline{a}}_{\mathrm{ee}} + 3\varepsilon_0\ \underline{\underline{I}}\right\}.\end{aligned} \tag{19d}$$

When (12a-d) are substituted into (19a-d), the resulting values of the constitutive parameters of the bianisotropic composite (14) are too cumbersome for inclusion here. Nevertheless, they constitute an extensive generalization of the Maxwell-Garnet formula given for composites made of isotropic dielectric spheres suspended in an isotropic dielectric host medium.

Before concluding, it may be of use to consider the limiting case $N(4\pi/3)a^3 \ll 0.1$. In such a dilute composite, the Lorentz fields $\{\mathbf{E}_{\mathrm{L}}, \mathbf{H}_{\mathrm{L}}\}$ can be simply replaced by $\{\mathbf{E}, \mathbf{H}\}$ in (16), because the spheres will not interact with each other. Expressions (19a-d) simplify considerably in that case, and are given by

$$\underline{\underline{\varepsilon}}_{\mathrm{eff}} = (N/\varepsilon_0)\ \underline{\underline{a}}_{\mathrm{ee}} + \underline{\underline{I}}, \qquad \underline{\underline{\alpha}}_{\mathrm{eff}} = (N/\varepsilon_0)\ \underline{\underline{a}}_{\mathrm{em}}. \tag{20a, b}$$

$$\underline{\underline{\mu}}_{\text{eff}} = (N/\mu_0)\ \underline{\underline{a}}_{\text{mm}} + \underline{\underline{I}}\ , \qquad \underline{\underline{\beta}}_{\text{eff}} = (N/\mu_0)\ \underline{\underline{a}}_{\text{me}}\ . \tag{20c, d}$$

Acknowledgements.

This work was supported in part by a Research Initiation Grant from the Pennsylvania State University.

References

[1] MIE G., Beitrage zür Optik trüber Medien speziell kolloidaler Metallösungen, *Ann. Phys.* **25** (1908) 377-445.
[2] DEBYE P., Der Lichtdruck auf Kugeln von beliebigem Material, *Ann. Phys.* **30** (1909) 57-136.
[3] BOHREN C.F., Light scattering by an optically active sphere, *Chem. Phys. Lett.* **29** (1974) 458-462.
[4] TAYLOR D.J., Scattering and extinction by anisotropic spheres, *J. Wave-Mater. Interact.* **3** (1988) 327-339.
[5] ROTH J. and DIGNAM M.J., Scattering and extinction cross sections for a spherical particle coated with an oriented molecular layer, *J. Opt. Soc. Am.* **63** (1973) 308-311.
[6] PURCELL E.M. and PENNYPACKER C.R., Scattering and absorption of light by non spherical dielectric grains, *Astrophys. J.* **186** (1973) 705-714.
[7] VARADAN V.V., LAKHTAKIA A. and VARADAN V.K., Scattering by three-dimensional anisotropic scatterers, *IEEE Trans. Antennas. Propagat.* **37** (1989) 800-802.
[8] VAN DE HULST H.C., Light Scattering by Small Particles (New-York: Wiley) 1957.
[9] PALMER B.R., STAMATAKIS P., BOHREN C.F. and SALZMAN G.C., A multiple-scattering model for opacifying particles in polymer films, *J. Coat. Technol.* **61** (1989) 41-47.
[10] JACKSON J.D., Classical Electrodynamics (New-York: Wiley) 1975.
[11] MAXWELL-GARNET J.C., Colours in metal glasses and in metallic films, *Philos. Trans. Roy. Soc. Lond. A* **203** (1904) 385-420.
[12] WARD L., The Optical Constants of Bulk Materials and Films (Bristol: Adam Hilger) 1988.
[13] BOHREN C.F. and HUFFMAN D.R., Absorption and Scattering of Light by Small Particles (New-York: Wiley) 1983.
[14] BRUGGEMANN D.A.G., Berechnung verschiedener physikalischer Konstanten von heterogenen Substanzen. I. Dielektrizitätskonstanten und Leitfähigkeiten der Mischkörper aus isotropen Substanzen, *Ann. Phys.* **24** (1935) 636-679.
[15] WACHNIEWSKI A. and MCCLUNG H.B., New approach to effective medium for composite materials: Application to electromagnetic properties, *Phys. Rev. B* **33** (1986) 8053-8059.
[16] DING K.H. and TSANG L., Effective propagation constants in media with densely distributed dielectric particles of multiple sizes and permittivities, in *Prog. Electromagn. Res. I*, Ed. J.A. Kong (New York: Elsevier) 1989.
[17] VARADAN V.K., BRINGI V.N., VARADAN V.V. and ISHIMARU A., Multiple scattering theory for waves in discrete random media and comparison with experiments, *Radio Sci.* **18** (1983) 321-327.
[18] MILGROM M. and SHTRIKMAN S., A layered-shell model of isotropic composites and exact expressions for the effective properties, *J. Appl. Phys.* **66** (1989) 3429-3436.
[19] POST E.J., Formal Structure of Electromagnetics (Amsterdam: North-Holland) 1962.
[20] KONG J.A., Theory of Electromagnetic Waves (New York: Wiley) 1975.
[21] LAKHTAKIA A., VARADAN V.K. and VARADAN V.V., Time-Harmonic Electromagnetic Fields in Chiral Media (Heidelberg: Springer-Verlag) 1989.
[22] SINGHAM S.B., Intrinsic optical activity in light scattering from an arbitrary particle, *Chem. Phys. Lett.* **130** (1986) 139-144.
[23] BORN M. and WOLF E., Principles of Optics (Oxford: Pergamon) 1987.
[24] Ed. A. Lakhtakia, Selected Papers on Natural Optical Activity (Bellingham, Washington, USA: SPIE Opt. Engg. Press) 1990.
[25] VAN BLADEL J., Relativity and Engineering (Berlin: Springer-Verlag) 1984.

[26] FEDOROV F.I., Theory of Gyrotropy (in Russian) (Minsk: Nauka i Teknika) 1976.
[27] JONES D.S., Low frequency electromagnetic radiation, *J. Inst. Math. Applics.* **23** (1979) 421-447.
[28] LAKHTAKIA A., VARADAN V.K. and VARADAN V.V., Dilute random distribution of small chiral spheres, *Appl. Opt.* **28** (1990) in press.
[29] STEVENSON A.F., Solution of electromagnetic scattering problems as power series in the ratio (dimension of scatterer)/(wavelength), *J. Appl. Phys.* **24** (1953) 1134-1142.
[30] VAN BLADEL J., Electromagnetic Fields (Washington, DC, USA: Hemisphere) 1985.
[31] SIHVOLA A.H. and LINDELL I.V., Chiral Maxwell-Garnet mixing formula, *Electron. Lett.* **26** (1990) 118-119.
[32] LORRAIN P. and CORSON D.R., Electromagnetic Fields and Waves (New York: Freeman) 1970.

Cet article a été imprimé avec le Macro Package "Editions de Physique Avril 1990".