Optik 84, No. 5 (1990) 160–162 © Wissenschaftliche Verlagsgesellschaft mbH, Stuttgart

On extending the Brewster law at planar interfaces

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Received November 13, 1989

Abstract

On extending the Brewster law at planar interfaces. The reflection of planewaves at planar chiral-uniaxial interfaces has been examined in order to broaden the concept of the Brewster law into what may be termed as the *Brewster reflection de-correlation condition.*

Inhalt

Erweiterung des Brewstergesetzes an planaren Grenzschichten. Die Reflexion von Planwellen an planaren chiral-einachsigen Grenzflächen wird untersucht, um das Konzept des Brewsterwinkels in Richtung *„Brewster Reflexions Dekorrelationsbedingung"* zu erweitern.

Introduction

In 1815, Sir David Brewster described [1] his experiments on the reflection of unpolarised light from planar dielectric-dielectric interfaces. Data collected by him gave rise to what is now called the *Brewster angle*, and was condensed by him into the *Brewster law*. Modern textbooks tend to give an un-Brewsterian definition of the Brewster law [2], which is more faithfully stated for dielectric-dielectric interfaces as: If unpolarized light is incident at this angle, the reflected light is plane-polarized. It is the purpose of this communication to broaden the concept of the Brewster angle into what may be termed as the *Brewster reflection de-correlation condition*. This will be done by examining the reflection of plane waves at planar chiral-uniaxial interfaces.

Theoretical Development

Consider the interface $z = 0$: a homogeneous, lossless, uniaxial dielectric medium occupies the half-space $z \geqq 0$; while the half-space $z \leqq 0$ is filled with an isotropic, homogeneous, lossless, chiral medium.

The chiral medium, characterized by [3]

$$\boldsymbol{D} = \varepsilon[\boldsymbol{E} + \beta \nabla \times \boldsymbol{E}]; \quad \boldsymbol{B} = \mu[\boldsymbol{H} + \beta \nabla \times \boldsymbol{H}], \tag{1}$$

is circularly birefringent. Thus, the fields in the region $z \leqq 0$ are circularly polarised, and may be represented using the vectors [3]

$$\begin{aligned}\boldsymbol{Q}_1 = {} & A_1[\boldsymbol{e}_y + i(-\delta_1 \boldsymbol{e}_x + \kappa \boldsymbol{e}_z)/\gamma_1] \exp[i(\kappa x + \delta_1 z)] \\ & + B_1[\boldsymbol{e}_y + i(\delta_1 \boldsymbol{e}_x + \kappa \boldsymbol{e}_z)/\gamma_1] \exp[i(\kappa x - \delta_1 z)]; \\ & z \leqq 0,\end{aligned} \tag{2a}$$

$$\begin{aligned}\boldsymbol{Q}_2 = {} & A_2[\boldsymbol{e}_y + i(\delta_2 \boldsymbol{e}_x - \kappa \boldsymbol{e}_z)/\gamma_2] \exp[i(\kappa x + \delta_2 z)] \\ & + B_2[\boldsymbol{e}_y - i(\delta_2 \boldsymbol{e}_x + \kappa \boldsymbol{e}_z)/\gamma_2] \exp[i(\kappa x - \delta_2 z)]; \\ & z \leqq 0.\end{aligned} \tag{2b}$$

Here, the wavenumbers are given by $\gamma_1 = k/(1 - k\beta)$ and $\gamma_2 = k/(1 + k\beta)$; $k = \omega\sqrt{(\mu\varepsilon)}$ is merely a shorthand notation; while $\delta_1 = +\sqrt{(\gamma_1^2 - \kappa^2)}$ and $\delta_2 = +\sqrt{(\gamma_2^2 - \kappa^2)}$. The coefficients A_1 and A_2 represent plane waves incident on the interface, while B_1 and B_2 denote the plane waves reflected off into the chiral half-space. The electromagnetic fields in this region are given by

$$\boldsymbol{E} = \boldsymbol{Q}_1 - i\eta \boldsymbol{Q}_2, \quad \boldsymbol{H} = \boldsymbol{Q}_2 - (i/\eta)\boldsymbol{Q}_1; \quad z \leqq 0, \tag{3}$$

with $\eta = \sqrt{(\mu/\varepsilon)}$. An $\exp[-i\omega t]$ time-dependence has been assumed, while κ is the horizontal wavenumber required by Snell's law to satisfy the phase-matching condition at the interface $z = 0$; and $\boldsymbol{e}_x$, etc., are the unit Cartesian vectors.

The constitutive relations for the unixial medium are specified as [4]

$$\boldsymbol{D} = \varepsilon_{\perp u}\boldsymbol{E} + (\varepsilon_{\parallel u} - \varepsilon_{\perp u})\,\boldsymbol{c}(\boldsymbol{c} \cdot \boldsymbol{E}); \quad \boldsymbol{B} = \mu_u \boldsymbol{H}, \quad z \geqq 0 \tag{4}$$

in which the optic axis is represented by the unit vector $\boldsymbol{c}$, in all generality [5], as

$$\boldsymbol{c} = \boldsymbol{e}_x \sin\xi + \boldsymbol{e}_z \cos\xi, 0^\circ \leqq \xi \leqq 180^\circ. \tag{5}$$

It is well known that the planewaves in the unixial medium are of the ordinary and the extraordinary types. Thus, an appropriate representation of the planewaves in this half-space can be set down as

$$\begin{aligned}E_x = {} & C_2 \exp[i(\kappa x + \delta_{2u+} z)] \\ & + D_2 \exp[i(\kappa x - \delta_{2u-} z)],\end{aligned} \tag{6a}$$

$$\begin{aligned}E_y = {} & C_1 \exp[i(\kappa x + \delta_{1u} z)] \\ & + D_1 \exp[i(\kappa x - \delta_{1u} z)],\end{aligned} \tag{6b}$$

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$$H_x = J_{1u}\{-C_1 \exp[i(\kappa x + \delta_{1u} z)] + D_1 \exp[i(\kappa x - \delta_{1u} z)]\}, \quad (6c)$$

$$H_y = J_{2u}\{C_2 \exp[i(\kappa x + \delta_{2u+} z)] - D_2 \exp[i(\kappa x - \delta_{2u-} z)]\}, \quad (6d)$$

$$H_z = (\kappa/\omega \mu_u) E_y, \quad (6e)$$

$$-\omega E_z = [\kappa H_y + \omega(\varepsilon_{\parallel u} - \varepsilon_{\perp u}) E_x \sin\xi \cos\xi] / \cdot [\varepsilon_{\perp u} \sin^2\xi + \varepsilon_{\parallel u} \cos^2\xi]. \quad (6f)$$

In these expressions, the various quantities used are given as follows:

$$\delta_{1u} = +\sqrt{[k_{\perp u}^2 - \kappa^2]}, \quad (7a)$$

$$[\varepsilon_{\perp u} \sin^2\xi + \varepsilon_{\parallel u} \cos^2\xi]\, \delta_{2u+} = -\kappa(\varepsilon_{\parallel u} - \varepsilon_{\perp u}) \sin\xi \cos\xi + \sqrt{[\varepsilon_{\parallel u}\varepsilon_{\perp u}(k_{\perp u}^2 \sin^2\xi + k_{\parallel u}^2 \cos^2\xi - \kappa^2)]}, \quad (7b)$$

$$[\varepsilon_{\perp u} \sin^2\xi + \varepsilon_{\parallel u} \cos^2\xi]\, \delta_{2u-} = \kappa(\varepsilon_{\parallel u} - \varepsilon_{\perp u}) \sin\xi \cos\xi + \sqrt{[\varepsilon_{\parallel u}\varepsilon_{\perp u}(k_{\perp u}^2 \sin^2\xi + k_{\parallel u}^2 \cos^2\xi - \kappa^2)]}, \quad (7c)$$

$$J_{1u} = \delta_{1u}/\omega \mu_u, \quad (7d)$$

$$J_{2u} = \omega\sqrt{[\varepsilon_{\parallel u}\varepsilon_{\perp u}]}/\sqrt{[k_{\perp u}^2 \sin^2\xi + k_{\parallel u}^2 \cos^2\xi - \kappa^2]}, \quad (7e)$$

$$k_{\perp u}^2 = \omega^2 \mu_u \varepsilon_{\perp u}, \quad (7f)$$

$$k_{\parallel u}^2 = \omega^2 \mu_u \varepsilon_{\parallel u}. \quad (7g)$$

The coefficients D_1 and D_2 represent plane waves incident on the interface, while C_1 and C_2 represent plane waves reflected off the interface.

The boundary value problem is solved by ensuring the continuity of the tangential components of the $\boldsymbol{E}$ and the $\boldsymbol{H}$ fields across the interface $z = 0$. For a given κ, the resulting solution is best stated in matrix notation as follows:

$$\begin{pmatrix} B_1 \\ B_2 \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} + \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix} \begin{pmatrix} D_1 \\ D_2 \end{pmatrix}, \quad (8a)$$

$$\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} + \begin{pmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{pmatrix} \begin{pmatrix} D_1 \\ D_2 \end{pmatrix}. \quad (8b)$$

The various Fresnel reflection and transmission coefficients involved in the foregoing matrices are given as follows:

$$\begin{aligned}
\Delta R_{11} &= -p_+ q_- + s_- & \Delta R_{22} &= p_+ q_- + s_- \\
\Delta R_{12} &= 2i\eta(\delta_2/\gamma_2)\, p_- & \Delta R_{21} &= -2i(\delta_1/\gamma_1 \eta)\, p_- \\
\Delta r_{11} &= p_- q_+ - s_- & \Delta R_{22} &= p_- q_+ + s_- \\
\Delta r_{12} &= -2i\eta J_{2u} q_- & \Delta r_{21} &= -2i\eta J_{1u} q_- \\
\Delta T_{11} &= 4(\delta_1/\gamma_1)\, u_2 & \Delta T_{22} &= 4\eta(\delta_2/\gamma_2)\, v_1 \\
\Delta T_{12} &= -4i\eta(\delta_2/\gamma_2)\, u_1 & \Delta T_{21} &= -4i(\delta_1/\gamma_1)\, v_2 \\
\Delta t_{11} &= 2\eta J_{1u} u_2 & \Delta t_{22} &= -2J_{2u} v_1 \\
\Delta t_{12} &= -2i\eta J_{2u} v_2 & \Delta t_{21} &= 2iJ_{1u} u_1
\end{aligned}$$

where

$$\begin{aligned}
\Delta &= p_+ q_+ + s_+ & p_\pm &= \eta^2 J_{1u} J_{2u} \pm 1 \\
q_\pm &= (\delta_2/\gamma_2) \pm (\delta_1/\gamma_1) & s_\pm &= 2\eta[(\delta_1/\gamma_1)(\delta_2/\gamma_2) J_{2u} \pm J_{1u}] \\
u_1 &= \eta(\delta_1/\gamma_1)\, J_{2u} + 1 & u_2 &= \eta(\delta_2/\gamma_2)\, J_{2u} + 1 \\
v_1 &= \eta J_{1u} + (\delta_1/\gamma_1) & v_2 &= \eta J_{1u} + (\delta_2/\gamma_2).
\end{aligned}$$

Analysis

Suppose now that $D_1 = D_2 = 0$, so that incidence is from the chiral side only. The condition on the horizontal wavenumber κ such that the ratio (B_1/B_2) is independent of the ratio (A_1/A_2) can be obtained easily following Chen [4], and is given by

$$R_{12} R_{21} = R_{11} R_{22}, \quad (9)$$

which can be succintly expressed as

$$p_+ q_+ - s_+ = 0. \quad (10)$$

Therefore, if κ satisfies eq. (10), the reflection ratio (B_1/B_2) is completely *decorrelated* from the incidence ratio (A_1/A_2).

Now, let $A_1 = A_2 = 0$, so that incidence is from the uniaxial side only. In order that the reflection ratio (C_1/C_2) be independent of the incidence ratio (D_1/D_2), the condition

$$r_{12} r_{21} = r_{11} r_{22}, \quad (11)$$

must be satisfied. But, eq. (11) also boils down to $p_+ q_+ - s_+ = 0$!

Thus, it is appropriate that eq. (10) be referred to as the *Brewster reflection decorrelation condition* for planar chiral-uniaxial interfaces, regardless of which half-space the incidence is from. This is a very general statement, since by setting $\beta = 0$, the chiral half-space can be made to be achiral; whereas by setting $\varepsilon_{\parallel u} = \varepsilon_{\perp u}$, the uniaxial half-space can be made to be isotropic. Hence, eq. (10) constitutes the chief result of this communication.

Consider also the case of normal incidence, i.e., $\kappa = 0$. Both r_{11} and r_{12} are directly proportional to q_-. Consequently, a normally-incident ordinary (resp. extraordinary) plane wave on the uniaxial side is reflected back as an ordinary (resp. extraordinary) plane wave. On the other hand, R_{11} and R_{12} are not zero when $\kappa = 0$. Hence, even for a normally incident left- (resp. right-) circularly polarized planewave, the reflected field will have both left- and right-circularly plarized components (unless $\xi = 0°$); this is a direct consequence of the anisotropy of the uniaxial medium.

Finally, the following two relationships should also be noted:

$$[(R_{12}/i\eta) + (i\eta R_{21})]\,[1 - R_{11}R_{22} + (R_{12}/i\eta)(i\eta R_{21})]^{-1} = p_-/p_+\,, \tag{12a}$$

$$[r_{11} - r_{22}]\,[1 - r_{11}r_{22} + r_{12}r_{21}]^{-1} = -\,s_-/s_+\,. \tag{12b}$$

These two relations are in the same vein as the relations between Fresnel reflection coefficients derived by Azzam [6] for planar dielectric-dielectric interfaces. Since the uniaxial medium is anisotropic, the left sides of both eqs. (12a) and (12b) contain κ; corresponding to the case investigated by Azzam, the left sides would not contain κ and be independent of the angle of incidence.

References

[1] D. B. Brewster, "On the laws which regulate the polarization of light by reflection from transparent bodies," Philos. Trans. Roy. Soc. Lond. **105** (1815) 105–129.

[2] A. Lakhtakia, "Would Brewster recognize today's Brewster angle?" Optics News **15** (6) (1989) 14–18.

[3] A. Lakhtakia, V. K. Varadan and V. V. Varadan, *Time-Harmonic Electromagnetic Fields in Chiral Media* (Springer-Verlag, Berlin, 1989).

[4] H. C. Chen, *Theory of Electromagnetic Waves* (Wiley, New York, 1983), p. 246.

[5] The optic axis of the uniaxial medium should be represented by $\boldsymbol{c} = \boldsymbol{e}_x \sin\xi\cos\zeta + \boldsymbol{e}_y \sin\xi\sin\zeta + \boldsymbol{e}_z \cos\xi\,\{0^\circ \leqq \xi \leqq 180^\circ, 0^\circ \leqq \zeta \leqq 360^\circ\}$, in all generality. However, a simple rotation of the co-ordinate system about the z-axis reduces $\boldsymbol{c}$ to the form given as eq. (5).

[6] R. M. A. Azzam, "Relationship between the p and s Fresnel reflection coefficients of an interface independent of angle of incidence," J. Opt. Soc. Am. A**3** (1986) 928–929.