PROPAGATION IN A PARALLEL-PLATE WAVEGUIDE WHOLLY FILLED WITH A CHIRAL MEDIUM

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ABSTRACT

Propgation of electromagnetic waves in a parallel-plate waveguide wholly filled with a chiral medium is examined. The dispersion equation derived leads to two sets of modes. Propagation constants for the two sets have been numerically obtained.

1. INTRODUCTION

Although the phenomenon of chirality is known chiefly at the molecular level, it has been suggested [1] that particles endowed with chirality can exist at even lower frequencies, say, in the GHz range. This is because chirality, or handedness, is a geometric property: for example, the electromagnetic response of a right-handed helix is different from that of a left-handed one [2]. Furthermore, by embedding such chiral particles in a low-loss dielectric medium, that medium too will possess handedness. With advances in polymer science, it is becoming increasingly possible that such artificial materials can be manufactured with ease, and their properties tailored by altering the sizes and concentration of the embedded chiral particles.

Significant advances have taken place recently in the formulation of a frequency-domain electromagnetic theory for chiral media, and these have been summarised by us elsewhere [1]. With these developments, it is time that aspects relating to the application of chiral media for practical problems be explored. To that end, we have already obtained the eigenmodes of a perfectly conducting sphere filled with a homogeneous, isotropic, chiral medium [3]. In continuation of our aim, we study here the modes of a parallel-plate waveguide filled with a chiral material. It is our conjecture that this geometry will be of use in the development of integrated circuitry with chiral substrates.

Consider a source-free region occupied by an isotropic chiral medium in which the usual constitutive relations $\mathbf{D} = \varepsilon\mathbf{E}$ and $\mathbf{B} = \mu\mathbf{H}$ do not hold due to their incompatibility with the handedness of the medium. Instead, the relations

$$\mathbf{D} = \varepsilon\mathbf{E} + \beta\varepsilon\,\nabla\times\mathbf{E} \quad ; \quad \mathbf{B} = \mu\mathbf{H} + \beta\mu\,\nabla\times\mathbf{H} \tag{1}$$

hold, and satisfy the requirements of time-reversal symmetry and reciprocity. Following [4], the electromagnetic field is transformed to

$$\mathbf{E} = \mathbf{Q}_1 + a_R\,\mathbf{Q}_2 \quad ; \quad \mathbf{H} = \mathbf{Q}_2 + a_L\,\mathbf{Q}_1 \quad , \tag{2}$$

where the left- (LCP) and the right- (RCP) circularly polarized fields, $\mathbf{Q}_1$ and $\mathbf{Q}_2$, respectively, must satisfy the Helmholtz equations

$$\{\nabla^2 + \gamma_1^2\}\,\mathbf{Q}_1 = 0 \quad ; \quad \{\nabla^2 + \gamma_2^2\}\,\mathbf{Q}_2 = 0 \quad , \tag{3}$$

along with the rotational conditions

$$\nabla\times\mathbf{Q}_1 = \gamma_1\,\mathbf{Q}_1 \quad ; \quad \nabla\times\mathbf{Q}_2 = -\gamma_2\,\mathbf{Q}_2 \quad . \tag{4}$$

Needless to say, these fields are also divergence-free, *vide*

$$\nabla\cdot\mathbf{Q}_1 = 0 \quad ; \quad \nabla\cdot\mathbf{Q}_2 = 0 \quad . \tag{5}$$

In these equations, the two wavenumbers are given by

$$\gamma_1 = k / \{1 - k\beta\} \quad ; \quad \gamma_2 = k / \{1 + k\beta\} \quad , \tag{6}$$

and

$$a_L = -i\,(\varepsilon/\mu)^{1/2} \quad ; \quad a_R = -i\,(\mu/\varepsilon)^{1/2} \quad . \tag{7}$$

An exp[$-i\omega t$] time dependence has been assumed throughout this work, while $k = \omega\{\mu\varepsilon\}^{1/2}$ is simply a shorthand notation.

2. MODAL ANALYSIS

Consider the region bounded by the perfectly conducting plates $z = \pm d$, which is wholly filled with a chiral medium. It is well-known that although LCP and RCP waves can propagate independently in an unbounded chiral region, at a bimaterial interface mode-conversion occurs [4]. Therefore, pure LCP or RCP modes cannot exist within a bounded chiral volume. Using a representation given by us earlier [5], the electromagnetic field inside the parallel-plate waveguide can be adequately expressed by

$$Q_1 = \Big[A_{1+}\,\gamma_1^{-1}\,\{-\alpha_1\,\mathbf{e}_x + \kappa\,\mathbf{e}_z - i\gamma_1\,\mathbf{e}_y\}\exp[i\alpha_1 z]$$

$$+ A_{1-}\,\gamma_1^{-1}\,\{\alpha_1\,\mathbf{e}_x + \kappa\,\mathbf{e}_z - i\gamma_1\,\mathbf{e}_y\}\exp[-i\alpha_1 z]\Big]\exp[i\kappa x] \quad , \tag{8a}$$

$$Q_2 = \Big[A_{2+}\,\gamma_2^{-1}\,\{-\alpha_2\,\mathbf{e}_x + \kappa\,\mathbf{e}_z + i\gamma_2\,\mathbf{e}_y\}\exp[i\alpha_2 z]$$

$$+ A_{2-}\,\gamma_2^{-1}\,\{\alpha_2\,\mathbf{e}_x + \kappa\,\mathbf{e}_z + i\gamma_2\,\mathbf{e}_y\}\exp[-i\alpha_2 z]\Big]\exp[i\kappa x] \quad , \tag{8b}$$

which satisfy the phase-matching conditions *via* the horizontal wavenumber κ. In these equations, $\alpha_1 = +\sqrt{(\gamma_1^2 - \kappa^2)}$ and $\alpha_2 = +\sqrt{(\gamma_2^2 - \kappa^2)}$, while $A_{1\pm}$ and $A_{2\pm}$ are the unknown field coefficients.

The boundary conditions require that the tangential components of the electric field be identically zero on the surfaces $z = \pm d$. The use of (2) and (8) along with the boundary conditions leads to the dispersion equation

$$0 = \Big[\{\alpha_1\gamma_1^{-1} + \alpha_2\gamma_2^{-1}\}\,\{1 - \exp[i2\,(\gamma_1 + \gamma_2)\,d]\} + \{\alpha_1\gamma_1^{-1} - \alpha_2\gamma_2^{-1}\}\,\{\exp[i2\gamma_1\,d]\} - \exp[i2\gamma_2\,d]\}\Big] \times$$

$$\Big[\{\alpha_1\gamma_1^{-1} + \alpha_2\gamma_2^{-1}\}\,\{1 - \exp[i2\,(\gamma_1 + \gamma_2)\,d]\} - \{\alpha_1\gamma_1^{-1} - \alpha_2\gamma_2^{-1}\}\,\{\exp[i2\gamma_1\,d]\} - \exp[i2\gamma_2\,d]\}\Big]. \tag{9}$$

Since (9) contains two factors on its right hand side, it is clear that two kinds of modes can exist. The dispersion equation for the *first* set of modes is given by

$$0 = \Big[\{\alpha_1\gamma_1^{-1} + \alpha_2\gamma_2^{-1}\}\,\{1 - \exp[i2\,(\gamma_1 + \gamma_2)\,d]\} + \{\alpha_1\gamma_1^{-1} - \alpha_2\gamma_2^{-1}\}\,\{\exp[i2\gamma_1\,d]\} - \exp[i2\gamma_2\,d]\}\Big] \, , \tag{10a}$$

and for these modes it is easily seen that

$$A_{1+} / A_{1-} = A_{2+} / A_{2-} = -1 \quad , \tag{10b}$$

$$a_R\,A_{2+} / A_{1+} = \sin(\alpha_1 d) / \sin(\alpha_2 d) \quad . \tag{10c}$$

As a consequence the field components E_x and H_x are *even* with respect to z, whereas the field components E_y, H_y, E_z and H_z are *odd* with respect to z.

On the other hand, the dispersion equation for the *second* set of modes is given by

$$0 = \left[\{ \alpha_1 \gamma_1^{-1} + \alpha_2 \gamma_2^{-1} \} \{ 1 - \exp[i2(\gamma_1 + \gamma_2) d] \} - \{ \alpha_1 \gamma_1^{-1} - \alpha_2 \gamma_2^{-1} \} \{ \exp[i2\gamma_1 d] \} - \exp[i2\gamma_2 d] \} \right] , \tag{11a}$$

and for these modes it can be shown that

$$A_{1+} / A_{1-} = A_{2+} / A_{2-} = 1 , \tag{11b}$$

$$a_R A_{2+} / A_{1+} = \cos(\alpha_1 d) / \cos(\alpha_2 d) . \tag{11c}$$

As a consequence the field components E_x and H_x are *odd* with respect to z, whereas the field components E_y, H_y, E_z and H_z are *even* with respect to z.

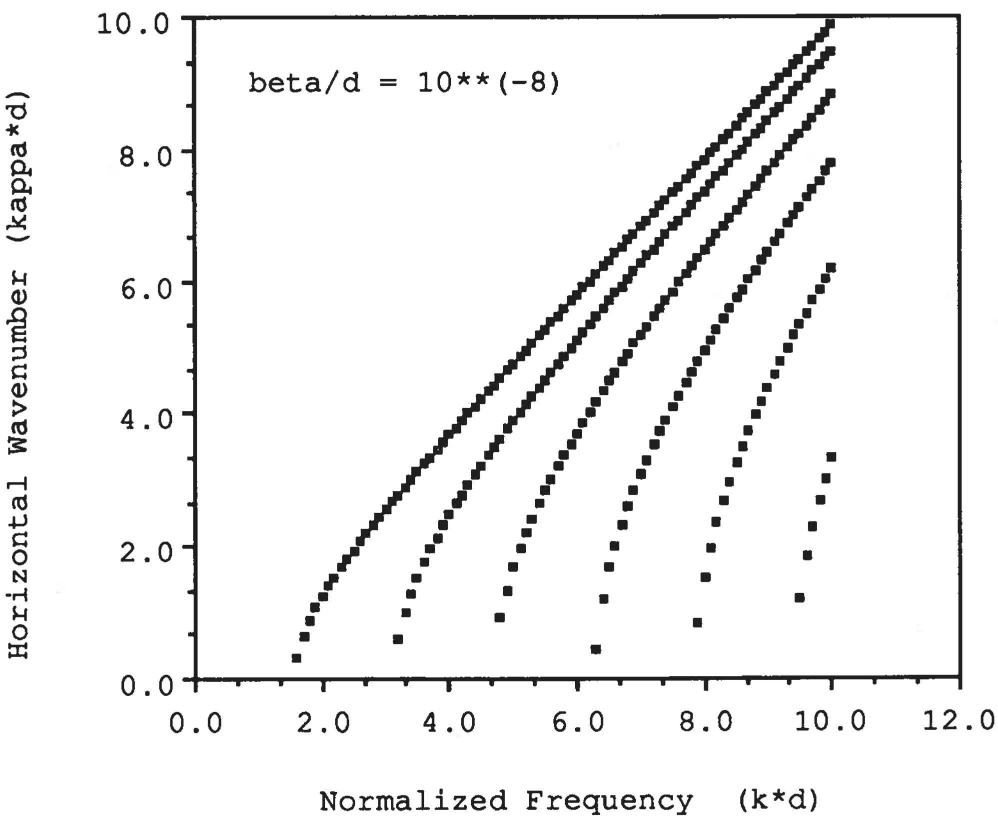

Figure 1. Solutions κd of the dispersion equations (12c) and (13c) are virtually identical for $\beta/d \le 10^{-4}$. They also correspond (almost exactly) to the TE- and TM- polarised fields when $\beta/d = 0$.

3. NUMERICAL RESULTS

With the developments of the previous section, the modal fields of the *first* set can be compactly set down as

$$\mathbf{Q}_1 = \exp[i\kappa x]\, \gamma_1^{-1} \{ -\alpha_1 \cos(\alpha_1 z)\, \mathbf{e}_x + i\kappa \sin(\alpha_1 z)\, \mathbf{e}_z + \gamma_1 \sin(\alpha_1 z)\, \mathbf{e}_y \} , \tag{12a}$$

$$a_R \mathbf{Q}_2 = \exp[i\kappa x]\, \gamma_2^{-1} \{ -\alpha_2 \cos(\alpha_2 z)\, \mathbf{e}_x + i\kappa \sin(\alpha_2 z)\, \mathbf{e}_z - \gamma_2 \sin(\alpha_2 z)\, \mathbf{e}_y \} \sin(\alpha_1 d) / \sin(\alpha_2 d) \quad ; \tag{12b}$$

the horizontal wavenumber κ can be determined for this set by solving the equation

$$\alpha_1\gamma_2 / \alpha_2\gamma_1 + \tan(\alpha_1 d) / \tan(\alpha_2 d) = 0 \quad . \tag{12c}$$

In a similar vein, the modal fields of the *second* set can be written as

$$Q_1 = \exp[i\kappa x]\, \gamma_1^{-1}\, \{-\alpha_1 \sin(\alpha_1 z)\, e_x + \kappa\cos(\alpha_1 z)\, e_z - i\gamma_1 \cos(\alpha_1 z)\, e_y\} \ , \tag{13a}$$

$$a_R\, Q_2 = \exp[i\kappa x]\, \gamma_2^{-1}\, \{-i\alpha_2 \sin(\alpha_2 z)\, e_x + \kappa\cos(\alpha_2 z)\, e_z + i\gamma_2 \cos(\alpha_2 z)\, e_y\} \cos(\alpha_1 d) / \cos(\alpha_2 d) \quad ; \tag{13b}$$

the horizontal wavenumber κ can be determined for this set by solving the equation

$$\alpha_1\gamma_2 / \alpha_2\gamma_1 + \cot(\alpha_1 d) / \cot(\alpha_2 d) = 0 \quad . \tag{13c}$$

The solutions κd of the dispersion equations (12c) and (13c) were obtained numerically on a Macintosh II minicomputer as functions of the normalized frequency $kd \le 10.0$ for various values of β/d; while $\kappa \le \min[\gamma_1, \gamma_2]$, $k\beta$ was kept less than 0.5 for Figs. 1-3, it is less than 0.99 for Figs. 4 and 5. Fig.1 shows the calculations for $\beta/d = 10^{-8}$. For $\beta/d \le 10^{-4}$, the roots for the two sets of modes did not appreciably differ from each other; furthermore $\alpha_1 d$ and $\alpha_2 d$ were approximately equal to integral multiples of $\pi/2$. The effect of chirality became numerically appreciable, however, at $\beta/d = 10^{-3}$, although the differences between the roots for the two sets are still small enough to be appreciated on a graph.

Shown in Figs. 2 and 3 are the roots κd of the dispersion equations (12c) and (13c), respectively, for $\beta/d = 10^{-2}$. The root structures are now different for the two sets, and particularly so when kd is high. When β/d increases even further, the differences are even more telling, as illustrated in Figs. 4 and 5. Thus it is only at the higher frequencies, and for a higher degree of chirality as characterised by larger values of $|\beta|$, that the effect of the chirality of the medium becomes significant.

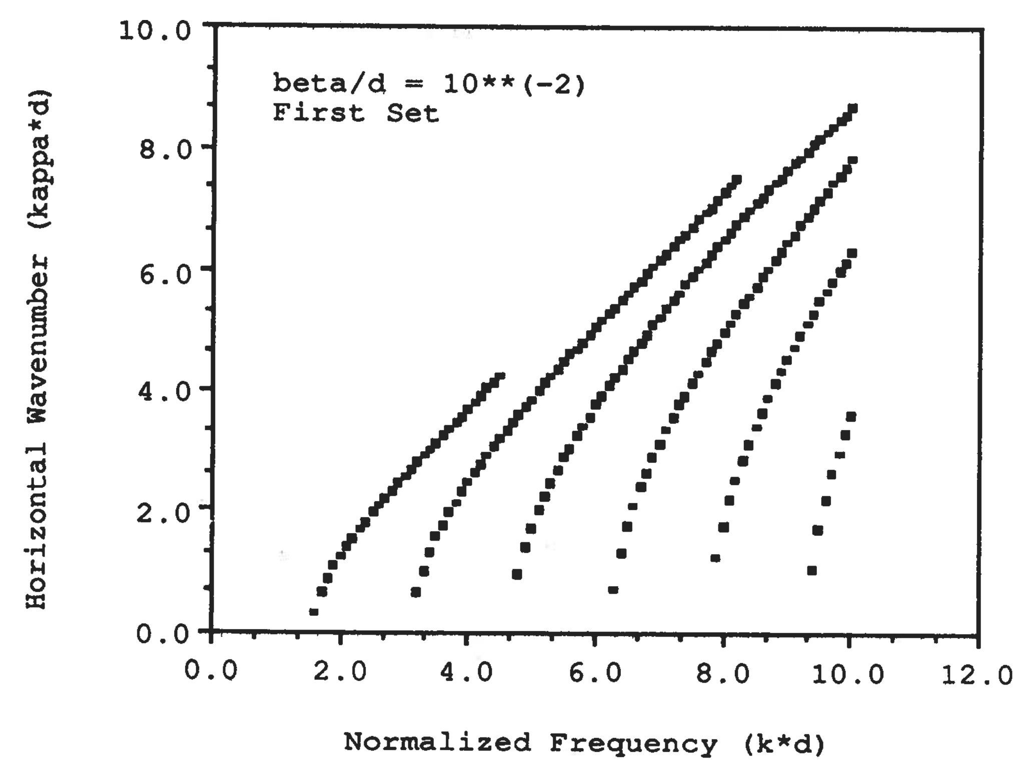

Figure 2. Solutions κd of the dispersion equation (12c) of the first set at $\beta/d = 10^{-2}$; $k\beta \le 0.5$.

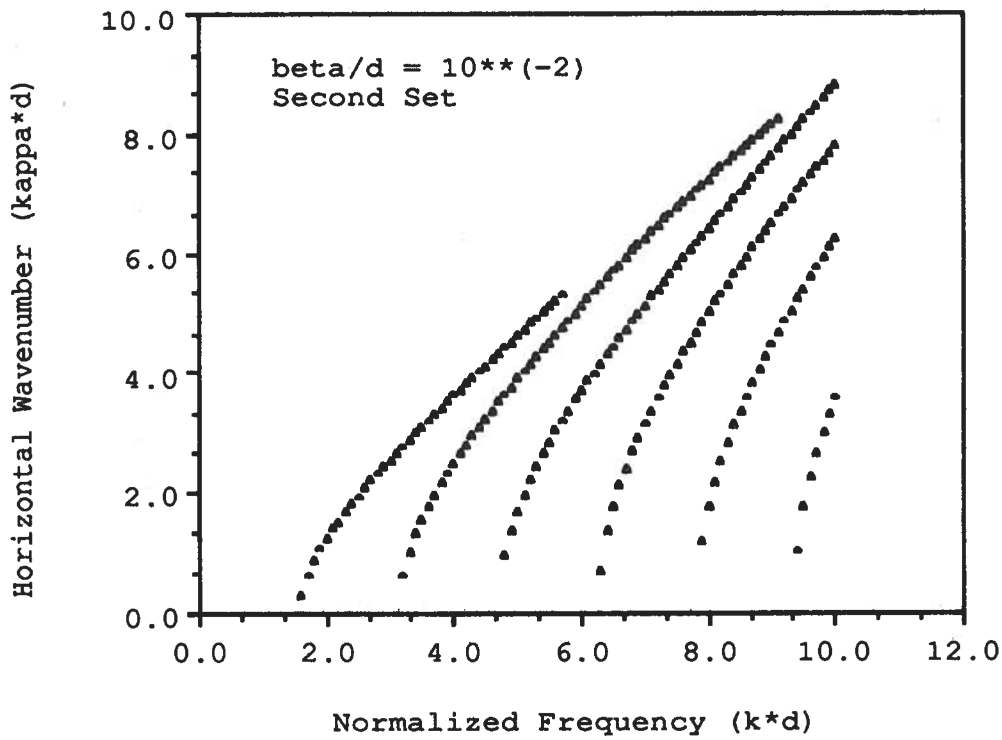

Figure 3. Solutions κd of the dispersion equation (13c) of the second set at $\beta/d = 10^{-2}$; $k\beta \leq 0.5$.

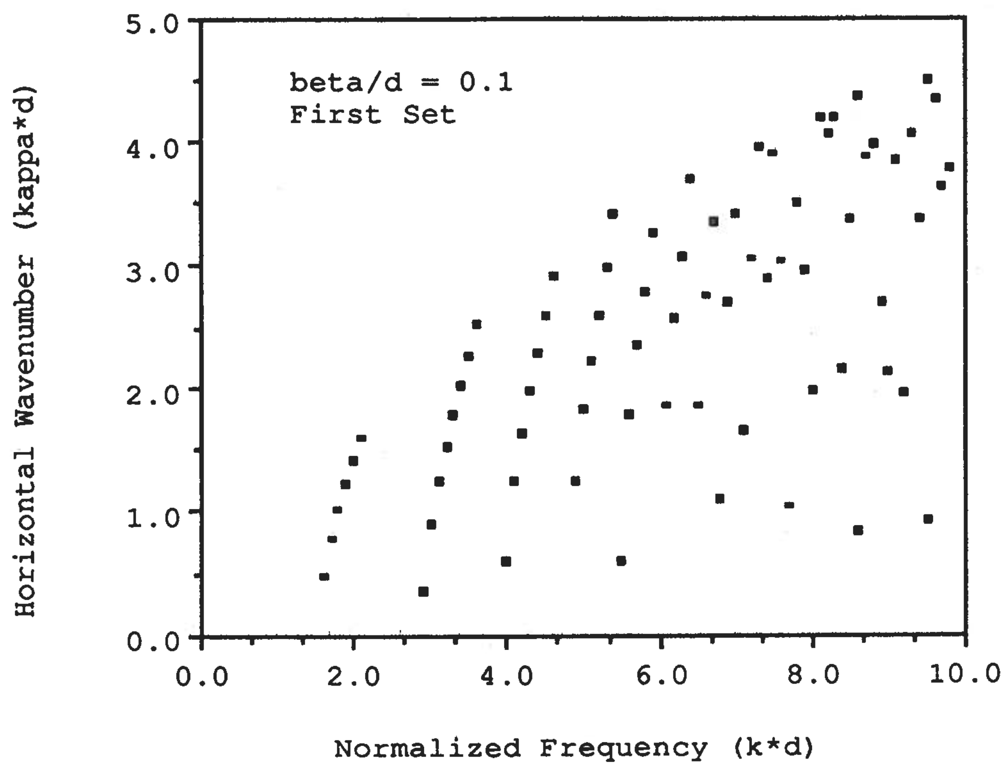

Figure 4. Solutions κd of the dispersion equation (12c) of the first set at $\beta/d = 10^{-1}$; $k\beta \leq 0.99$.

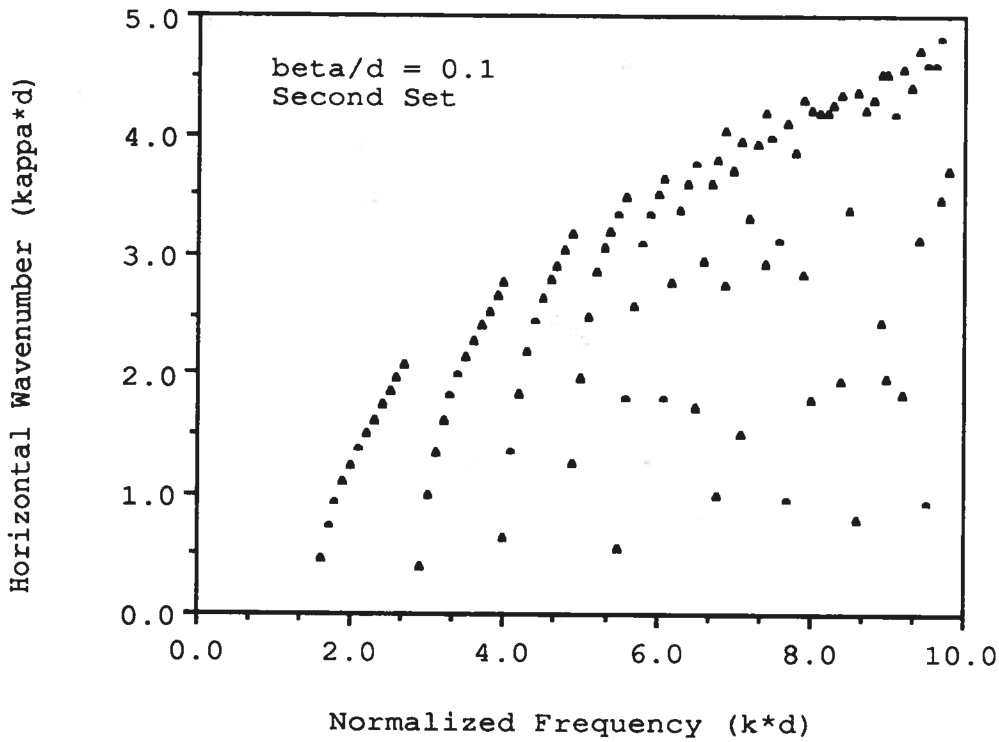

Figure 5. Solutions κd of the dispersion equation (13c) of the second set at $\beta/d = 10^{-1}$; $k\beta \leq 0.99$.

ACKNOWLEDGEMENTS

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Errata:

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Equations (9), (10a), (11a) and (13a) of the subject paper [1] contained typographical errors. The correct versions are as follows:

$$0 = \left[\{\alpha_1\gamma_1^{-1} + \alpha_2\gamma_2^{-1}\}\,\{1 - \exp[i2(\alpha_1 + \alpha_2)d]\} + \{\alpha_1\gamma_1^{-1} - \alpha_2\gamma_2^{-1}\}\,\{\exp[i2\alpha_1 d]\} - \exp[i2\alpha_2 d]\}\right] \times \left[\{\alpha_1\gamma_1^{-1} + \alpha_2\gamma_2^{-1}\}\,\{1 - \exp[i2(\alpha_1 + \alpha_2)d]\} - \{\alpha_1\gamma_1^{-1} - \alpha_2\gamma_2^{-1}\}\,\{\exp[i2\alpha_1 d]\} - \exp[i2\alpha_2 d]\}\right]. \tag{9}$$

$$0 = \left[\{\alpha_1\gamma_1^{-1} + \alpha_2\gamma_2^{-1}\}\,\{1 - \exp[i2\,(\alpha_1 + \alpha_2)d]\} + \{\alpha_1\gamma_1^{-1} - \alpha_2\gamma_2^{-1}\}\,\{\exp[i2\alpha_1 d]\} - \exp[i2\alpha_2 d]\}\right], \tag{10a}$$

$$0 = \left[\{\alpha_1\gamma_1^{-1} + \alpha_2\gamma_2^{-1}\}\,\{1 - \exp[i2\,(\alpha_1 + \alpha_2)d]\} - \{\alpha_1\gamma_1^{-1} - \alpha_2\gamma_2^{-1}\}\,\{\exp[i2\alpha_1 d]\} - \exp[i2\alpha_2 d]\}\right], \tag{11a}$$

$$Q_1 = \exp[i\kappa x]\,\gamma_1^{-1}\,\{-i\alpha_1 \sin(\alpha_1 z)\,e_x + \kappa\cos(\alpha_1 z)\,e_z - i\gamma_1 \cos(\alpha_1 z)\,e_y\}\,, \tag{13a}$$

REFERENCE

1. V.K. Varadan, V.V. Varadan and A. Lakhtakia, 'Propagation in a parallel-plate waveguide wholly filled with a chiral medium', *J. Wave-Mater. Interact.* 3, 267 (1988).