EXCITATION OF A PLANAR ACHIRAL/CHIRAL INTERFACE BY NEAR FIELDS

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ABSTRACT

The refraction of near fields by a planar achiral/chiral interface has been examined. A realistic source, a constant current line source, has been used here, as apart from the usual analyses involving incident planewaves only. Provided a planewave spectral decomposition of the incident field is possible, this procedure can be extended to include sources of other configurations and polarizations as well. Maps of the refracted field are drawn to elucidate the effect of both the near-zone irradiation as well as of the handedness of the chiral medium. Such an analysis would be of use to various researchers in the area so that the effect of the near fields of radiating sources may not be ignored.

1. INTRODUCTION

Ever since the discovery of optical activity early in the last century, there has been a great interest in measuring the circular dichroic (CD) and the optical rotatory dispersion (ORD) spectra of molecular aggregations [1]. Such measurements are routinely made, at frequencies down into the infra-red regime nowadays. The measurement procedures usually are variants of a simple technique: A planar slab of the optically active, or chiral, material is irradiated by a source, the polarization state of the irradiating field being known. On the other side of the slab, the rotation of the plane of polarization of the transmitted field with respect to that of the incident field is measured, and the CD and the ORD of the material determined. A good description of such an experiment conducted at frequencies around 10 GHz is available in [2], where Tinoco and Freeman describe the procedure for measuring the optical rotatory activity of a collection of oriented copper helices, each approximately 0.5 cm in diam and about 1 cm long.

However, this and other such measurement techniques usually ignore the proximity of the chiral slab to the source. If the source is far away from the exposed face of the slab, the illuminating field may be conveniently taken to be a planewave. But when that is not so, the slab lies in the near-zone of the source, where the irradiating field has reactive components of large magnitudes. In such circumstances, the planewave approximation of the source field can be quite gross. In this communication, the nature of the fields excited in a chiral half-space are examined. The source is taken to be a constant current line source, the field radiated by which source is decomposed into an infinite set of planewaves, some of which are propagating and the remaining ones are evanescent. It is to be emphasized here that any other source can be accommodated in this theoretical procedure, provided its field can be expressed by a planewave spectral (PWS) representation [3]. By mapping the field refracted into the chiral half-space, some understanding of the effect of near-field irradiation of achiral/chiral interfaces is obtained.

2. REFRACTION OF A TE-POLARIZED PLANEWAVE BY A PLANE ACHIRAL/CHIRAL INTERFACE

Let the space $z \leq 0$ be filled by a non-dissipative achiral medium (here taken to be free-space) in which the constitutive relations

$$D = \varepsilon_o E \quad , \quad B = \mu_o H \tag{1a,b}$$

hold, and from which a TE-polarized planewave

$$\mathbf{E}_i(\kappa) = \mathbf{j} \exp[j\{\kappa x + \beta_o(\kappa) z\}] \quad , \quad \mathbf{H}_i(\kappa) = (1/j\omega\mu_o) \nabla \times \mathbf{E}_i(\kappa) \quad , \tag{2a,b}$$

with

$$\beta_o(\kappa) = +[k_o^2 - \kappa^2]^{1/2} \quad , \quad k_o = \omega[\mu_o \varepsilon_o]^{1/2} \quad , \tag{3a,b}$$

is incident on the interface z = 0, the harmonic time-dependence exp[-jωt] being suppressed here and hereafter, and the unit vectors **i**, **j**, **k** having their usual meanings in a rectangular co-ordinate system.

The region above the z = 0 interface, z ≥ 0, is occupied by a chiral medium in which the constitutive equations are given as [4,5]

$$\mathbf{D} = \varepsilon\{\mathbf{E} + \alpha \nabla \times \mathbf{E}\} \quad , \quad \mathbf{B} = \mu\{\mathbf{H} + \alpha \nabla \times \mathbf{H}\} \tag{4a,b}$$

and in which it would be proper, because of Snell's law, to express the transmitted field as [6]:

$$\mathbf{E}_t(\kappa) = T_L(\kappa)\, \mathbf{Q}_L(\kappa) + a_R\, T_R(\kappa)\, \mathbf{Q}_R(\kappa) \quad ; \tag{5a}$$

$$\mathbf{H}_t(\kappa) = a_L\, T_L(\kappa)\, \mathbf{Q}_L(\kappa) + T_R(\kappa)\, \mathbf{Q}_R(\kappa) \tag{5b}$$

in which $T_{L,R}(\kappa)$ are to be determined by the boundary conditions on the interface z = 0, and the left- and the right-circularly polarized (LCP and RCP) wavefunctions, respectively, are given as

$$\mathbf{Q}_L(\kappa) = (1/k_L)\{-\beta_L(\kappa)\mathbf{i} - jk_L\mathbf{j} + \kappa\mathbf{k}\} \exp[j\{\kappa x + \beta_L(\kappa) z\}] \quad , \tag{6a}$$

and

$$\mathbf{Q}_R(\kappa) = (1/k_R)\{-\beta_R(\kappa)\mathbf{i} - jk_R\mathbf{j} + \kappa\mathbf{k}\} \exp[j\{\kappa x + \beta_R(\kappa) z\}] \quad . \tag{6b}$$

In Eqs. (5) and (6) the following definitions hold:

$$a_L = -j[\varepsilon/\mu]^{1/2} \quad , \quad a_R = -j[\mu/\varepsilon]^{1/2} \quad , \tag{7a,b}$$

$$k_L = k\,[1 - k\alpha]^{-1} \quad , \quad k_R = k\,[1 + k\alpha]^{-1} \quad , \tag{7c,d}$$

$$\beta_L(\kappa) = +[k_L^2 - \kappa^2]^{1/2} \quad , \quad \beta_R(\kappa) = +[k_R^2 - \kappa^2]^{1/2} \quad , \tag{7e,f}$$

$$k = \omega[\mu\varepsilon]^{1/2} \quad . \tag{7g}$$

The field reflected back into the achiral medium also has LCP and RCP components, and it can be expressed in the form [6]:

$$\mathbf{E}_r(\kappa) = R_R(\kappa)\, \mathbf{P}_R(\kappa) + R_L(\kappa)\, \mathbf{P}_L(\kappa) \quad , \quad \mathbf{H}_r(\kappa) = (1/j\omega\mu_o) \nabla \times \mathbf{E}_r(\kappa) \quad , \tag{8a,b}$$

where $R_{L,R}(\kappa)$ are to be determined and the wavefunctions,

$$\mathbf{P}_R(\kappa) = (1/k_o)\{\beta_o(\kappa)\mathbf{i} + jk_o\mathbf{j} + \kappa\mathbf{k}\} \exp[j\{\kappa x - \beta_o(\kappa) z\}] \quad , \tag{9a}$$

and

$$\mathbf{P}_L(\kappa) = (1/k_o)\{\beta_o(\kappa)\mathbf{i} - jk_o\mathbf{j} + \kappa\mathbf{k}\} \exp[j\{\kappa x - \beta_o(\kappa) z\}] \quad . \tag{9b}$$

By invoking the usual boundary conditions across the z = 0 interface, i.e.,

$$\mathbf{k} \times [\mathbf{E}_i(\kappa) + \mathbf{E}_r(\kappa) - \mathbf{E}_t(\kappa)] = 0 \quad , \quad \mathbf{k} \times [\mathbf{H}_i(\kappa) + \mathbf{H}_r(\kappa) - \mathbf{H}_t(\kappa)] = 0 \quad , \tag{10a,b}$$

and using Eqs. (1), (5) and (8), it can be shown that

$$T_L(\kappa) = 2j\,[S + a_R\, Q(\kappa)] / \Delta(\kappa) \quad , \tag{11a}$$

$$T_R(\kappa) = 2j\,[a_L\,S - P(\kappa)]\,/\,\Delta(\kappa) \quad , \tag{11b}$$

$$R_L(\kappa) = -(1/4)\,[T_L(\kappa)\,\{P(\kappa)+1\}\,\{a_L\,S+1\} + T_R(\kappa)\,\{Q(\kappa)-1\}\,\{S+a_R\}] \quad , \tag{11c}$$

$$R_R(\kappa) = (1/4)\,[T_L(\kappa)\,\{P(\kappa)-1\}\,\{a_L\,S-1\} + T_R(\kappa)\,\{Q(\kappa)+1\}\,\{S-a_R\}] \quad , \tag{11d}$$

$$\Delta(\kappa) = [S + a_R\,Q(\kappa)]\,[1 - a_L\,P(\kappa)\,S] - [a_L\,S - P(\kappa)]\,[a_R + Q(\kappa)\,S] \quad , \tag{11e}$$

and

$$P(\kappa) = (k_o/k_L)\,[\beta_L(\kappa)/\beta_o(\kappa)] \quad , \quad Q(\kappa) = (k_o/k_R)\,[\beta_R(\kappa)/\beta_o(\kappa)] \quad , \quad S = -j\,[\mu_o/\varepsilon_o]^{1/2} \quad . \tag{11f,g,h}$$

It should be noted that if $\alpha = 0$, i.e., the medium above the $z = 0$ interface also becomes achiral, then

$$k_R = k_L = k \quad , \quad \beta_R(\kappa) = \beta_L(\kappa) = \beta(\kappa) = +\,[k^2 - \kappa^2]^{1/2} \quad , \tag{12a,b}$$

$$P(\kappa) = Q(\kappa) = (k_o/k)\,[\beta(\kappa)/\beta_o(\kappa)] \quad , \tag{12c}$$

$$T_L(\kappa) = -\,a_R\,T_R(\kappa) = j\,a_R\,[a_R + Q(\kappa)\,S]^{-1} \quad , \tag{12d}$$

and

$$R_R(\kappa) = -\,R_L(\kappa) = (1/2)j\,[a_R - Q(\kappa)\,S]\,[a_R + Q(\kappa)\,S]^{-1} \quad ; \tag{12e}$$

consequently, the familiar expressions available in any standard textbook [e.g., 7] on EM theory emerge:

$$\mathbf{E}_t(\kappa) = \mathbf{j}\,2a_R\,[a_R + Q(\kappa)\,S]^{-1}\,\exp[j\,\{\kappa x + \beta(\kappa)z\}] \quad , \tag{13a}$$

and

$$\mathbf{E}_r(\kappa) = -\,\mathbf{j}\,[a_R - Q(\kappa)\,S]\,[a_R + Q(\kappa)\,S]^{-1}\,\exp[j\,\{\kappa x - \beta_o(\kappa)z\}] \quad . \tag{13b}$$

3. REFRACTION OF THE FIELD OF A LINE SOURCE

Let now an isotropically radiating, y-directed current line source be located at $\mathbf{r}_p = -\mathbf{k}d_p$ in the achiral half-space, and its radiated field be set as [8]:

$$\mathbf{E}_I = \mathbf{j}\pi H_0\,(k|\mathbf{r} - \mathbf{r}_p|) \quad . \tag{14}$$

A planewave spectral (PWS) decomposition [3] of this field is possible, and is given by

$$\mathbf{E}_I = \mathbf{j}\int_{-\infty}^{\infty} [d\kappa/\beta_o(\kappa)]\,\exp[j\beta_o(\kappa)d_p]\,\exp[j\,\{\kappa x + \beta_o(\kappa)z\}] \quad , \quad z \geq -d_p \quad , \tag{15a}$$

and

$$\mathbf{E}_I = \mathbf{j}\int_{-\infty}^{\infty} [d\kappa/\beta_o(\kappa)]\,\exp[-j\beta_o(\kappa)d_p]\,\exp[j\,\{\kappa x - \beta_o(\kappa)z\}] \quad , \quad z < -d_p \quad . \tag{15b}$$

In view of the PWS decomposition, Eq. (15a) and the development of the preceding section, it is possible to write down the total field existing in the chiral half-space due to the line source as

$$\mathbf{E}_T = \int_{-\infty}^{\infty} [d\kappa/\beta_o(\kappa)] \exp[j\beta_o(\kappa)d_p] \cdot \{T_L(\kappa)\, \mathbf{Q}_L(\kappa) + a_R\, T_R(\kappa)\, \mathbf{Q}_R(\kappa)\} \quad , \quad z > 0 \ . \quad (16)$$

Likewise, provided it is assumed that the field reflected from the achiral/chiral interface does not influence the line source current, the total field existing in the space $z < -d_p$ can be set down as

$$\mathbf{E}_R = \int_{-\infty}^{\infty} [d\kappa/\beta_o(\kappa)] \exp[j\beta_o(\kappa)d_p] \cdot \{R_R(\kappa)\, \mathbf{P}_R(\kappa) + R_L(\kappa)\, \mathbf{P}_L(\kappa)\} +$$

$$+ \mathbf{j} \int_{-\infty}^{\infty} [d\kappa/\beta_o(\kappa)] \exp[-j\beta_o(\kappa)d_p] \exp[j\{\kappa x - \beta_o(\kappa) z\}] \quad , \quad z < -d_p \quad . \quad (17)$$

4. DISCUSSION

It is the function $\mathbf{E}_T$ which has to be mapped in the xz plane in order to understand the fields generated in the chiral half-space. It is obvious that the κ-domain poles can be extracted simply by solving the equation $\Delta(\kappa) = 0$. However, such poles are of little interest in the present analysis. Instead, the only recourse is to actually map $\mathbf{E}_T$ over an xz grid for a given set of parameters. It would certainly be useful then to exploit the x- and the κ-symmetries of the various parts of the integrands in Eq. (16), and it turns out that

$$\mathbf{E}_T = 2\int_0^{\infty} [d\kappa/k_L\beta_o(\kappa)] \exp[j\beta_o(\kappa)d_p] \exp[j\beta_L(\kappa)z]\, T_L(\kappa)\, \{-\mathbf{i}\beta_L(\kappa)\cos\kappa x - \mathbf{j}jk_L\cos\kappa x + \mathbf{k}j\kappa\sin\kappa x\} +$$

$$+ 2a_R \int_0^{\infty} [d\kappa/k_R\beta_o(\kappa)] \exp[j\beta_o(\kappa)d_p] \exp[j\beta_R(\kappa)z]\, T_R(\kappa)\, \{-\mathbf{i}\beta_R(\kappa)\cos\kappa x + \mathbf{j}jk_R\cos\kappa x + \mathbf{k}j\kappa\sin\kappa x\} \ ,$$
$$z > 0 \ , \quad (18)$$

whose cartesian components have either even or odd symmetries with respect to the x coordinate.

A computer program to calculate $\mathbf{E}_T$ was implemented on a DEC VAX 11/730 minicomputer via Eq. (18). The infinite κ-integral in Eq. (18) was truncated to hold over the range $0 \le \kappa/k_o \le 20.0$, it being observed that this truncation did not give rise to any errors for the selected values $\varepsilon/\varepsilon_o \le 5.0$, $\mu/\mu_o = 1.0$, $\alpha k \le 0.1$ and $k_o d_p = 2.0$. The magnitudes of the x-, y-, and z- directed components of the LCP and the RCP parts of $\mathbf{E}_T$, as well as of the total $\mathbf{E}_T$, are illustrated in Figs. 1 - 4 for several cases over the rectangular region $0 \le kx/d_p, kz/d_p \le 10.0$.

In Fig. 1, $\mathbf{E}_T = \mathbf{E}_I$ since $\varepsilon/\varepsilon_o = 1.0$ and $\alpha k = 0.0$. This illustration, therefore, simply shows the field radiated by the line source in the specified xz domain when all space is covered by the non-dissipative achiral medium. As was expected and can be observed from this figure, the magnitudes of the RCP and the LCP parts of $\mathbf{E}_T$ are identical, but $\mathbf{E}_T$ is only y-directed. In Fig. 2, the ratio $\varepsilon/\varepsilon_o$ was increased to 5.0 with αk still equal to zero. Again, $\mathbf{E}_T$ turns out to have only a y-directed component, the x- and the z- directed components of its LCP and RCP parts having cancelled themselves out. These two figures are then simply symbolic of what can be expected when the refracting half-space is also achiral [8].

This cancellation of the x- and the z- directed components of the LCP and the RCP parts of $\mathbf{E}_T$ does not occur in Fig. 3 where $\varepsilon/\varepsilon_o = 5.0$ and $\alpha k = 0.01$. The magnitudes of the LCP and the RCP parts of $\mathbf{E}_T$ are still equal by virtue of the fact that all possible values of κ (positive as well as negative) are included in Eq. (18). However, the sum of these parts still shows the presence of the x- and the z- directed components, which had cancelled out in Figs. 1 and 2 where $\alpha k = 0.0$. The effect of handedness in the half-space $z \ge 0$ is betrayed, thus, by the presence of a transmitted electric field $\mathbf{E}_T$ which is not TE polarised like the source field $\mathbf{E}_I$. This tendency is even more marked in Fig. 4 where $\varepsilon/\varepsilon_o = 5.0$ and $\alpha k = 0.1$. Furthermore, in Fig. 4, $\mathbf{E}_T$ records well-defined extrema, which are not as prominent in Fig. 3. It becomes possible to state,

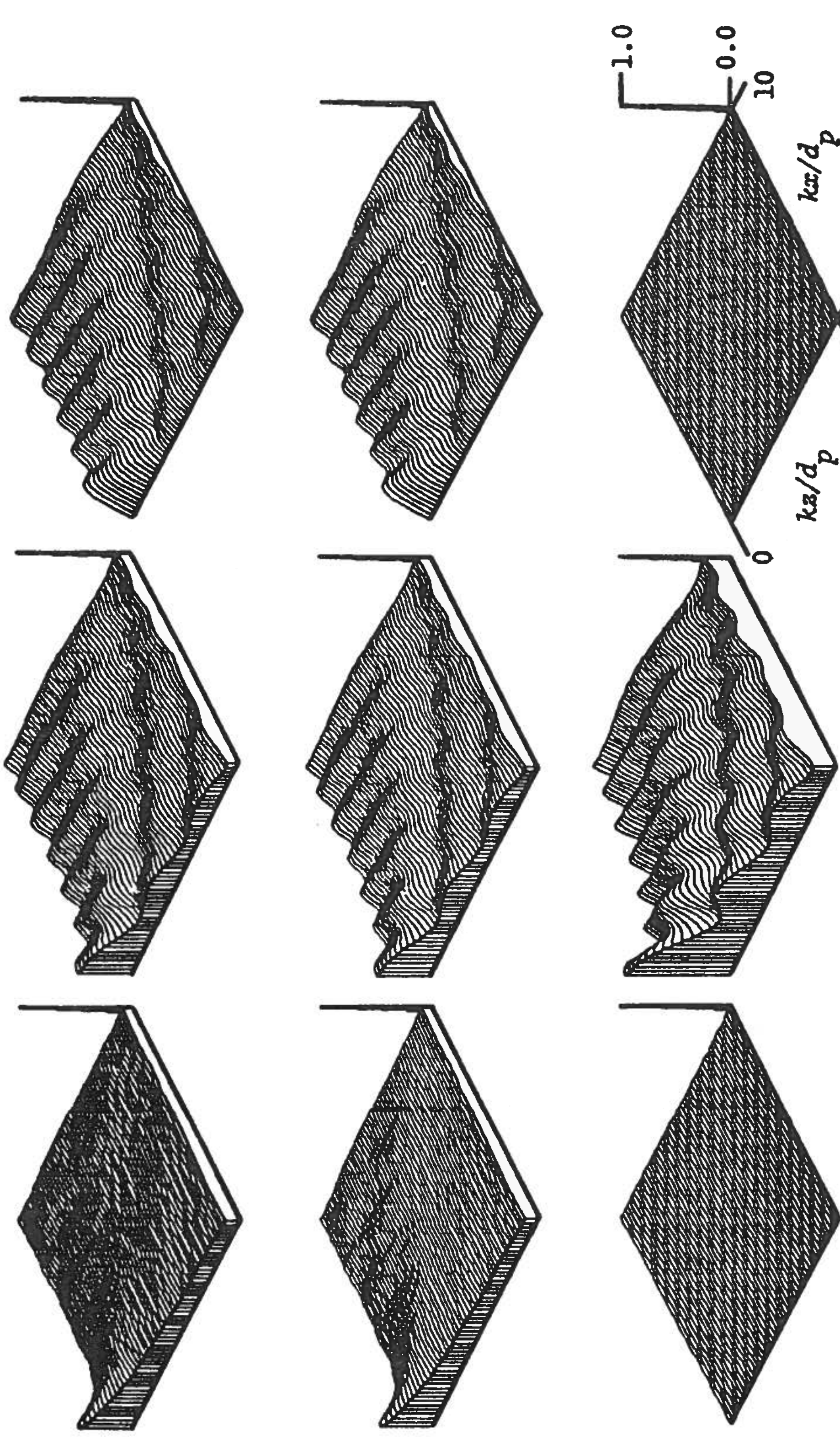

Figure 1 Magnitudes of the various components of E_T of Eq. (18) when E_I is given by Eq. (14). From left to right in each row: magnitudes of the x-, y- and z- directed components. From top to bottom: LCP part of E_T, RCP part of E_T, and E_T itself. The parameters $k_0 d_p = 2.0$, $\varepsilon/\varepsilon_0 = 1.0$, $\mu/\mu_0 = 1.0$ and $\alpha k = 0.0$. The field maps are drawn on the xz domain $0 \le kx/d_p, kz/d_p \le 10.0$.

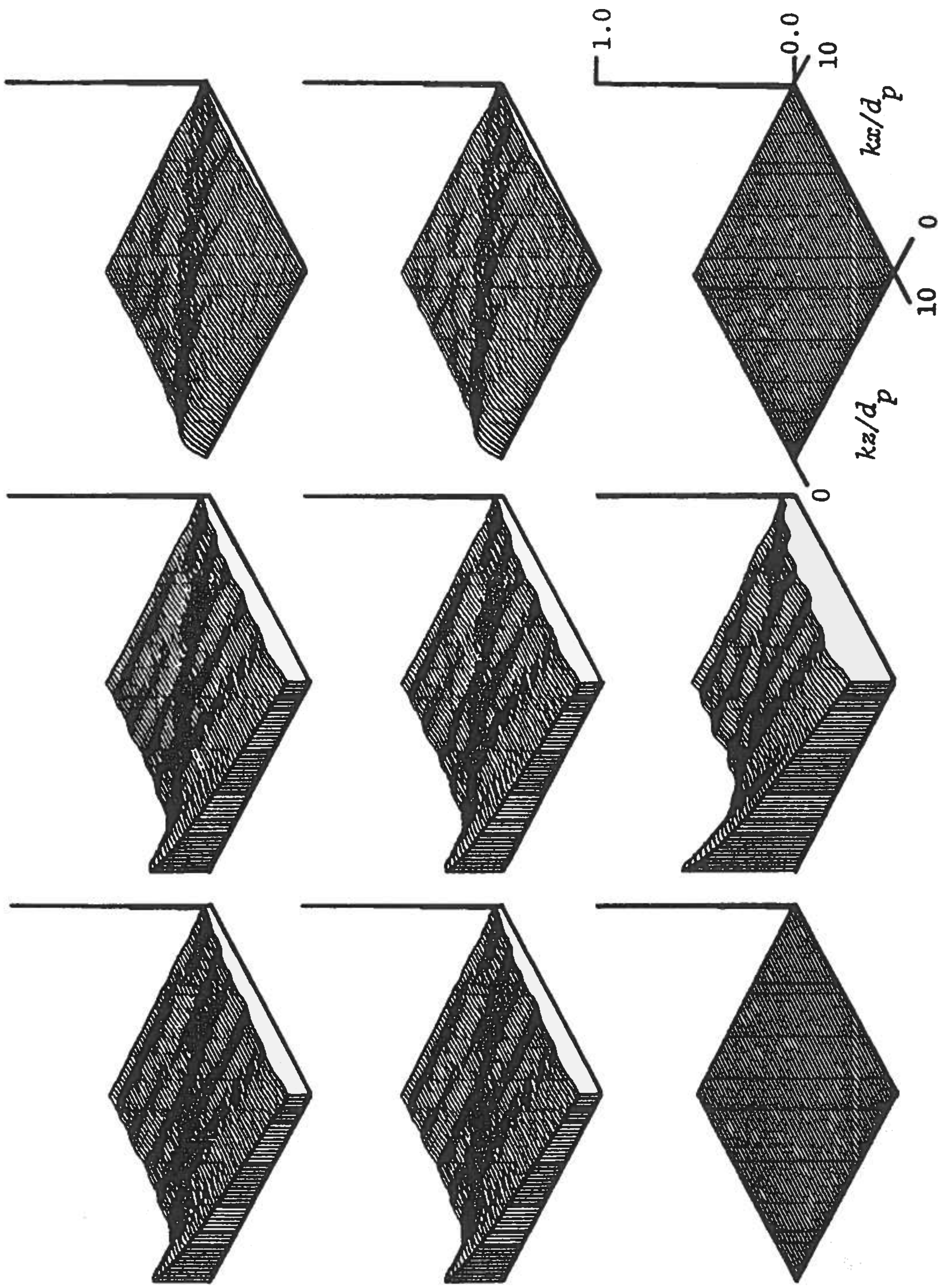

Figure 2 Same as Fig. 1, except $\varepsilon/\varepsilon_0 = 5.0$.

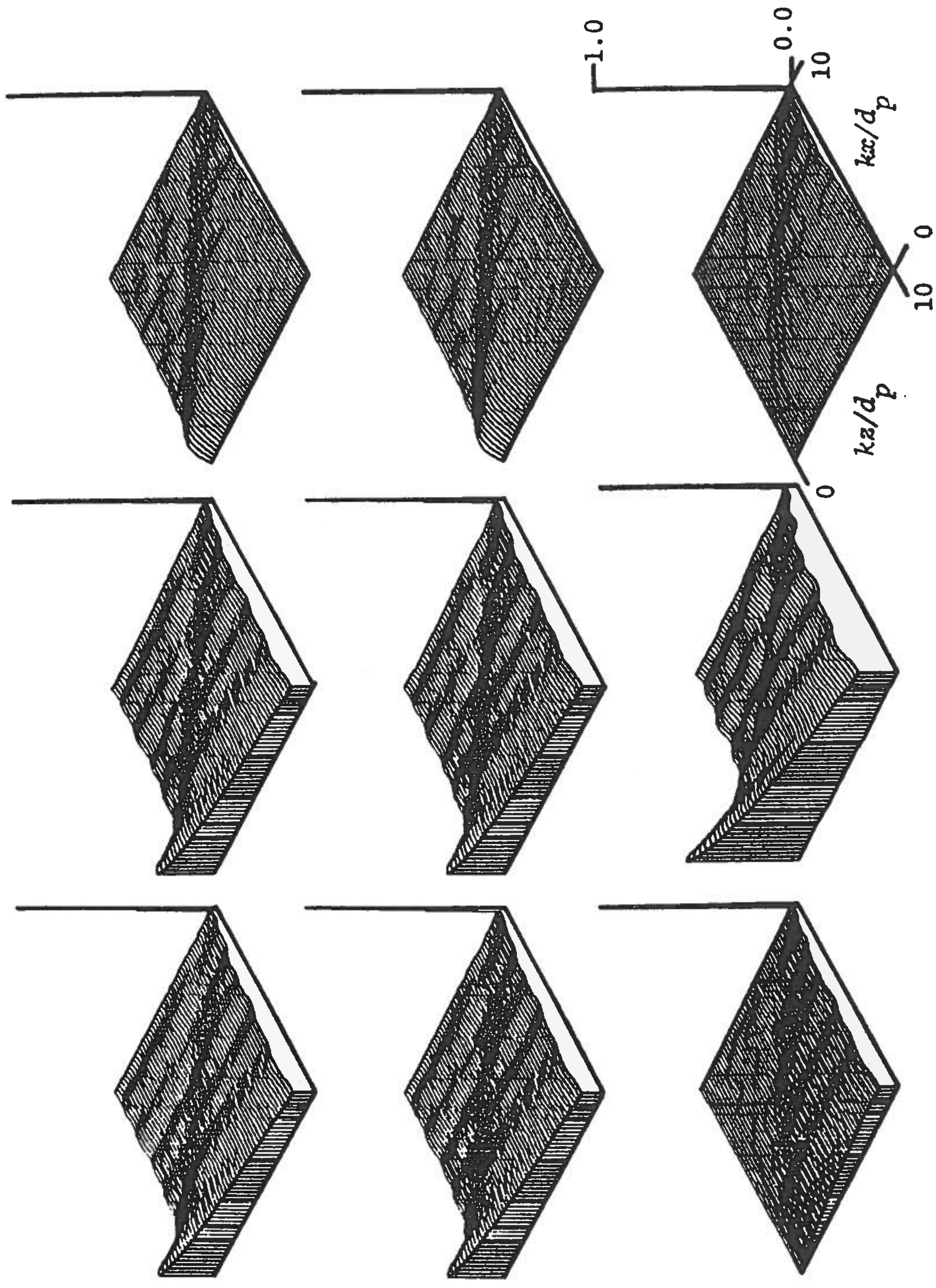

Figure 3 Same as Fig. 1, except $\varepsilon/\varepsilon_0 = 5.0$ and $\alpha k = 0.01$.

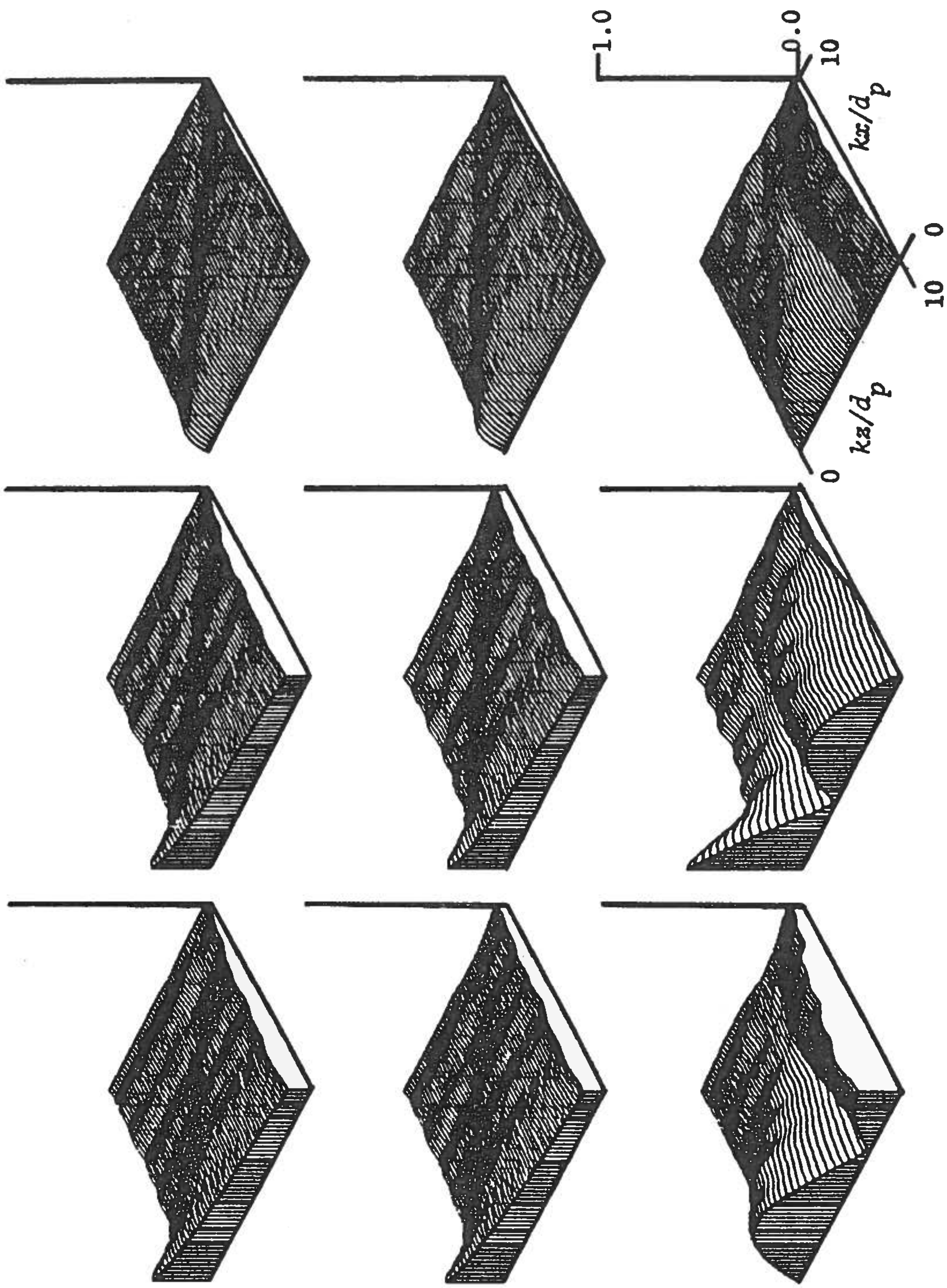

Figure 4 Same as Fig. 1, except $\varepsilon/\varepsilon_0 = 5.0$ and $\alpha k = 0.1$.

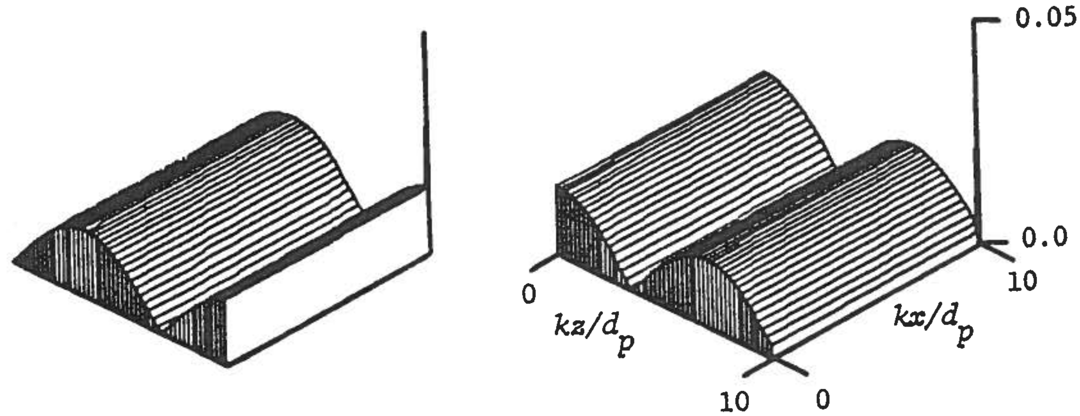

Figure 5 Magnitudes, from left to right, of the x- and y- directed components of $\mathbf{E}_T$ when $\mathbf{E}_I$ is given by Eq. (19). The parameters $k_o d_p = 2.0$, $\varepsilon/\varepsilon_o = 5.0$, $\mu/\mu_o = 1.0$ and $\alpha k = 0.1$. The field maps are drawn on the xz domain $0 \le kx/d_p$, $kz/d_p \le 10.0$.

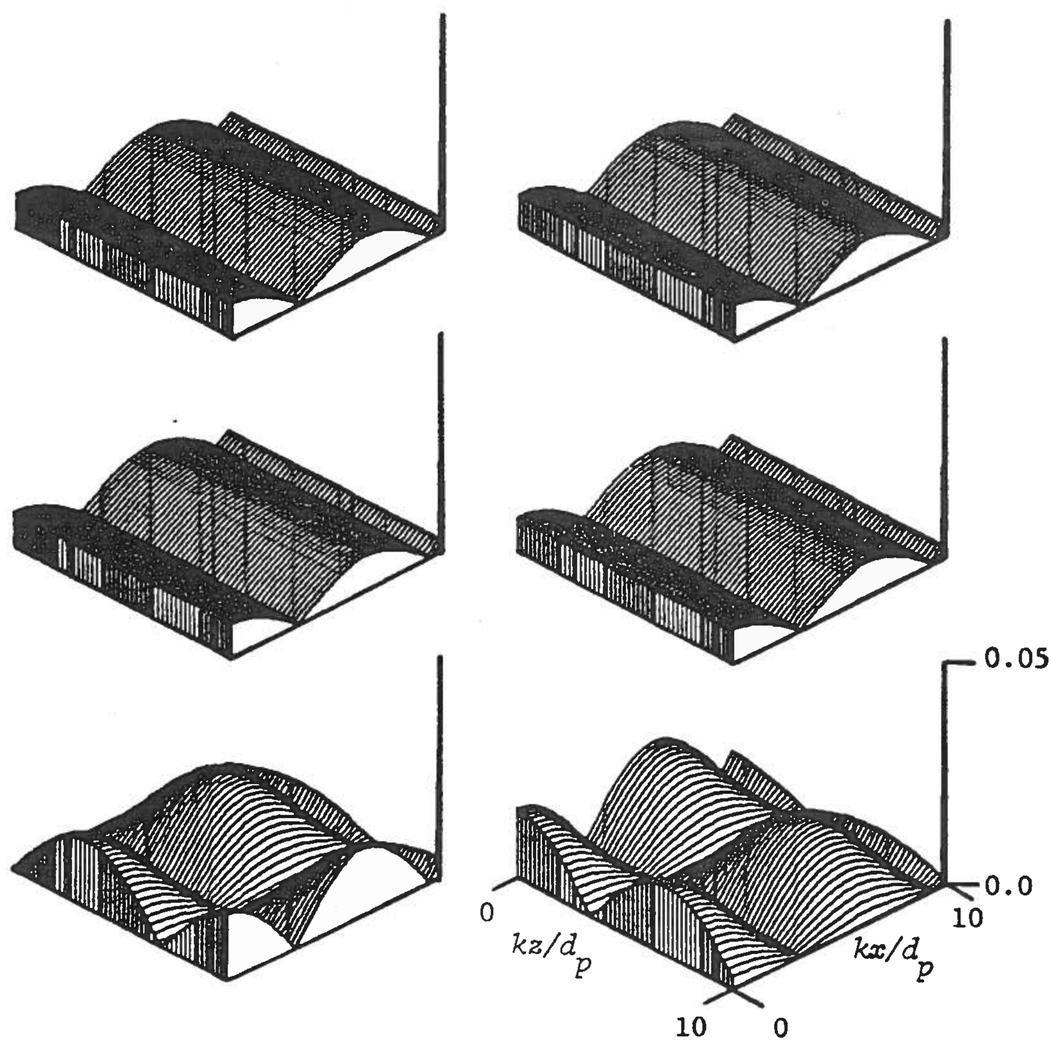

Figure 6 Magnitudes of the various components of $\mathbf{E}_T$ when $\mathbf{E}_I$ is given by Eq. (21). From left to right in each row: magnitudes of the x- and y- directed components. From top to bottom: LCP part of $\mathbf{E}_T$, RCP part of $\mathbf{E}_T$, and $\mathbf{E}_T$ itself. The parameters $k_o d_p = 2.0$, $\varepsilon/\varepsilon_o = 5.0$, $\mu/\mu_o = 1.0$ and $\alpha k = 0.1$. The field maps are drawn on the xz domain $0 \le kx/d_p$, $kz/d_p \le 10.0$.

therefore, that if $\alpha k << 1.0$, then $k_R \approx k_L \approx k$ and the effect of the chirality parameter α is slight on the field $\mathbf{E}_T$ as can be seen from comparing Fig. 3 with Fig. 2. But as αk increases, then k_R and k_L differ widely from k [9], and the interference of the LCP and the RCP parts of $\mathbf{E}_T$ is all too visible.

Some idea of this interference can be drawn from considering only specific values of κ in Eq. (18]) or, equivalently, by modifying the line source field $\mathbf{E}_I$ of Eq. (14) to

$$\mathbf{E}_I = j\pi H_0(k|\mathbf{r} - \mathbf{r}_p|)\,\delta(\kappa) \quad , \qquad (19)$$

where $\delta(\bullet)$ is the Dirac delta function. Shown in Fig. 5 are the magnitudes of the x- and the y- directed components of $\mathbf{E}_T$, with $\varepsilon/\varepsilon_0 = 5.0$ and $\alpha k = 0.1$, there being no z-directed component of $\mathbf{E}_T$. If Eq. (19])is used as the incident field, then, from the preceding theoretical analysis, it is easy to see that

$$\mathbf{E}_T = [2/k_o(S + a_R)]\exp[j(kz - k_o d_p)]\ \{-\mathbf{i}\sin(k^2\alpha z) + \mathbf{j}\cos(k^2\alpha z)\} \quad , \qquad (20a)$$

provided $\alpha k < 1$ and the approximations

$$k_L \approx k\,[1 + \alpha k] \quad ; \quad k_R \approx k\,[1 - \alpha k] \qquad (20b,c)$$

are valid. Fig. 5 vindicates Eq. (20a) very well and it also becomes easy to observe the mutual interference of the LCP and the RCP parts of $\mathbf{E}_T$ in deriving this equation.

Finally, in Fig. 6, the incident field is modified to

$$\mathbf{E}_I = j\pi H_0(k|\mathbf{r} - \mathbf{r}_p|)\,\delta(\kappa - k_o/4) \quad , \qquad (21)$$

and $\mathbf{E}_T$ is computed; $\varepsilon/\varepsilon_0 = 5.0$ and $\alpha k = 0.1$. It turns out that $P(k_o/4) \approx Q(k_o/4)$, while $\beta_L(k_o/4)/k_L \approx \beta_R(k_o/4)/k_R$. Consequently, $T_L(k_o/4) \approx -a_R T_R(k_o/4)$. Hence, the RCP and the LCP parts of $\mathbf{E}_T$ are virtually identical in magnitude, their z-directed components are negligible, and their x- and y- directed components have a $\cos(k_o x/4)$ dependence on the x coordinate. When, the LCP and the RCP parts are added up, the cartesian components of $\mathbf{E}_T$ have well-defined extrema in the xz plane. In view of Figs. 5 and 6, then, the variations of $\mathbf{E}_T$ in Fig. 4 can be easily explained.

It should be noted that $\mathbf{E}_T(x,y)$ in Figs. 1 - 4 tends to decay away as one goes farther from the source at $\mathbf{r}_p$. This is natural since $\mathbf{E}_I$, in the vicinity of the source, has reactive components with large magnitudes. As one moves farther and farther away from the source, these reactive, near-zone field components tend to die out. This behavior of $\mathbf{E}_I$ is replicated qualitatively by $\mathbf{E}_T$ as well, a phenomenon which has been noted in other electromagnetic scattering problems also [10].

In summary, it has been shown here that the fields refracted into a chiral half-space are markedly different when αk is substantially different from zero, i.e., when αk is of the order of 0.1. A realistic source has been used here, as apart from the usual analyses involving incident planewaves only [6]. Provided a PWS decomposition [3] of the incident field is possible, this procedure can be extended to include sources of other configurations and polarizations as well. Such an analysis would be of use to various researchers in the area so that the effect of the near fields [10] of radiating sources may not be ignored.

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