TIME-DEPENDENT DYADIC GREEN'S FUNCTIONS FOR UNIAXIAL DIELECTRIC MEDIA

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ABSTRACT

Causal, time-dependent Green's functions have been derived for lossless, uniaxial dielectric media, and their properties investigated. The obtained Green's functions are then used for the solution of the initial value problem, as well as for obtaining the radiated fields of time-varying, spatially concentrated sources.

1. INTRODUCTION

With the recent proliferation in the use of anisotropic materials for a variety of applications, researchers have begun to focus attention on the electromagnetic theory of such media. A reasonable foundation for this work was laid around a quarter of a century ago by Post [1], and his work was later extended by Kong and Cheng [2,3] and Krowne [4]. Missing from these and other works have generally been the time-harmonic and the time-dependent Green's functions, in useful and exact forms, which permit "easy" solutions of radiation and scattering problems.

In this paper we will concentrate on lossless, uniaxial dielectric media. The three-dimensional, time-harmonic electric Green's function for these media have been derived by Chen [5,6]. Because electromagnetic problems frequently require the utilization of magnetic sources, we have recently added to Chen's analysis by deriving the magnetic Green's function for the time-harmonic case [7]. Problems of practical interest, however, involve the radiation, propagation or scattering of time-limited signals, which can be neither periodic nor analytic functions of time. Hence, the development of causal, time-dependent Green's functions is of great importance. Therefore, in the sequel we have derived the time-dependent Green's functions for homogeneous, uniaxial dielectrics, and explored their characteristics. The derived Green's functions have then been used for the solution of the initial value problem, as well as for obtaining the electromagnetic fields radiated by some elementary, spatially concentrated sources.

2. TIME-DEPENDENT DYADIC GREEN'S FUNCTIONS

A homogeneous, uniaxial dielectric medium is characterized by one optic axis, i.e., its relative permittivity can be expressed in dyadic notation as [6]

$$\boldsymbol{\epsilon} = \varepsilon_{\perp} \mathbf{I} + (\varepsilon_{\parallel} - \varepsilon_{\perp})\, \mathbf{e}_c \mathbf{e}_c . \tag{1a}$$

Here, $\mathbf{I}$ is the idempotent, $\mathbf{e}_c$ is the unit vector parallel to the optic axis, the dyadic inverse of $\boldsymbol{\epsilon}$ is given as

$$\boldsymbol{\epsilon}^{-1} = (1/\varepsilon_{\perp})\, \mathbf{I} + [(1/\varepsilon_{\parallel}) - (1/\varepsilon_{\perp})]\mathbf{e}_c \mathbf{e}_c , \tag{1b}$$

and it is assumed that both $\varepsilon_{\perp}$ and $\varepsilon_{\parallel}$ are real and constant. Since $\mathbf{D}(\mathbf{r},t) = \varepsilon_0 \boldsymbol{\epsilon} \cdot \mathbf{E}(\mathbf{r},t)$ and $\mathbf{B}(\mathbf{r},t) = \mu_0 \mathbf{H}(\mathbf{r},t)$ are assumed, the last two Maxwell's equations are given as

$$\nabla \times \mathbf{E}(\mathbf{r}, t) = -\mu_0 \{\partial/\partial t\}\, \mathbf{H}(\mathbf{r}, t) - \mathbf{K}(\mathbf{r},t), \tag{2a}$$

$$\nabla \times \mathbf{H}(\mathbf{r},t) = \varepsilon_o \mathfrak{e} \cdot \{\partial/\partial t\}\, \mathbf{E}(\mathbf{r},t) + \mathbf{J}(\mathbf{r},t), \tag{2b}$$

in which ε_o and μ_o refer to free space; **J** and **K**, respectively, are the electric and the magnetic source current densities. After some manipulation of (2a,b), it can be seen that the time-dependent electric and magnetic fields obey the following partial differential equations:

$$[\nabla \times \nabla \times \mathfrak{I} + \mu_o \varepsilon_o \{\partial^2/\partial t^2\}\, \mathfrak{e}] \cdot \mathbf{E}(\mathbf{r},t) = -\mu_o \{\partial/\partial t\}\, \mathbf{J}(\mathbf{r},t) - \nabla \times \mathbf{K}(\mathbf{r},t), \tag{3}$$

$$[\nabla \times \mathfrak{e}^{-1} \cdot \nabla \times \mathfrak{I} + \mu_o \varepsilon_o \{\partial^2/\partial t^2\}\, \mathfrak{I}] \cdot \mathbf{H}(\mathbf{r},t) = -\varepsilon_o \{\partial/\partial t\} \mathbf{K}(\mathbf{r},t) + \nabla \times [\mathfrak{e}^{-1} \cdot \mathbf{J}(\mathbf{r},t)]. \tag{4}$$

The solutions of these two equations are sought in the forms

$$\mathbf{E}(\mathbf{r},t) = \iiint d^3 r_s \int dt_s [\mathfrak{G}_1(\mathbf{r},\mathbf{r}_s,t,t_s) \cdot \mathbf{J}(\mathbf{r}_s,t_s) + \mathfrak{G}_2(\mathbf{r},\mathbf{r}_s,t,t_s) \cdot \mathbf{K}(\mathbf{r}_s,t_s)], \tag{5}$$

$$\mathbf{H}(\mathbf{r},t) = \iiint d^3 r_s \int dt_s [\mathfrak{G}_3(\mathbf{r},\mathbf{r}_s,t,t_s) \cdot \mathbf{J}(\mathbf{r}_s,t_s) + \mathfrak{G}_4(\mathbf{r},\mathbf{r}_s,t,t_s) \cdot \mathbf{K}(\mathbf{r}_s,t_s)], \tag{6}$$

which satisfy the principle of superposition, and where $\mathfrak{G}_1$, etc., are the time-dependent Green's functions in dyadic notation. In this paper, it should be noted that the vector operations begin from the right-most one and proceed leftwards; boldface letters represent vectors; $\mathbf{e}_\zeta$, etc., are unit vectors; and the Gothic letters denote dyads.

The time-harmonic electric Green's function has been derived by Chen [5,6], and the corresponding magnetic Green's function has been derived by us elsewhere [7]. Thus, on assuming the time-dependence exp[-iωt] and noting that the medium is spatially invariant, it is easy to see that (3) and (4) can be transformed, respectively, to

$$\mathbf{E}(\mathbf{r},\omega) = \iiint d^3 r_s [\mathfrak{G}_e(\mathbf{r}-\mathbf{r}_s,\omega) \cdot [i\omega\mu_0 \mathbf{J}(\mathbf{r}_s,\omega)] - \{\mathfrak{e}^{-1} \cdot \nabla \times \mathfrak{G}_m(\mathbf{r}-\mathbf{r}_s,\omega)\} \cdot \mathbf{K}(\mathbf{r}_s,\omega)], \tag{7}$$

$$\mathbf{H}(\mathbf{r},\omega) = \iiint d^3 r_s [\{\nabla \times \mathfrak{G}_e(\mathbf{r}-\mathbf{r}_s,\omega)\} \cdot \mathbf{J}(\mathbf{r}_s,\omega) + i\omega\varepsilon_o\, \mathfrak{G}_m(\mathbf{r}-\mathbf{r}_s,\omega) \cdot \mathbf{K}(\mathbf{r}_s,\omega)]. \tag{8}$$

The time-harmonic electric and the magnetic Green's functions, respectively, are given as

$$4\pi\sqrt{\varepsilon_\perp}\; \mathfrak{G}_e(\mathbf{R},\omega) = [\varepsilon_\perp \varepsilon_\parallel \mathfrak{e}^{-1} + \nabla\nabla/k^2]\, g_e + \mathfrak{X}(\mathbf{R},\omega), \tag{9}$$

$$4\pi\sqrt{\varepsilon_\perp}\; \mathfrak{G}_m(\mathbf{R},\omega) = \varepsilon_\perp \{[\varepsilon_\perp \mathfrak{I} + \nabla\nabla/k^2]\, g_o - \mathfrak{X}(\mathbf{R},\omega)\}, \tag{10}$$

while

$$\mathfrak{X}(\mathbf{R},\omega) = (\varepsilon_\perp g_o - \varepsilon_\parallel g_e)\, [\,(\mathbf{R} \times \mathbf{e}_c)\, (\mathbf{R} \times \mathbf{e}_c) / (\mathbf{R} \times \mathbf{e}_c)^2]$$

$$+ \ (R_o g_o - R_e g_e)\, [\mathfrak{I} - \mathbf{e}_c \mathbf{e}_c - 2(\mathbf{R} \times \mathbf{e}_c)\, (\mathbf{R} \times \mathbf{e}_c) / (\mathbf{R} \times \mathbf{e}_c)^2] / ik\, (\mathbf{R} \times \mathbf{e}_c)^2, \tag{11}$$

$$4\pi\sqrt{(\varepsilon_\perp)}\; \nabla \times \mathfrak{G}_e(\mathbf{R},\omega) = \varepsilon_\perp (g_e - g_o)\, (\mathbf{R} \cdot \mathbf{e}_c)\, [\mathbf{e}_c \times (\mathbf{R} \times \mathbf{e}_c)][\mathbf{R} \times \mathbf{e}_c] / (\mathbf{R} \times \mathbf{e}_c)^4 +$$

$$+ \varepsilon_\perp (g_e - g_o)\, (\mathbf{R} \cdot \mathbf{e}_c)\, [\mathbf{R} \times \mathbf{e}_c]\, [\mathbf{e}_c \times (\mathbf{R} \times \mathbf{e}_c)] / (\mathbf{R} \times \mathbf{e}_c)^4 -$$

$$-\varepsilon_\perp^2 g_o R_o^{-2} (1 - ikR_o) [\mathbf{R} \times (\mathbf{R} \times \mathbf{e}_c)][\mathbf{R} \times \mathbf{e}_c] / (\mathbf{R} \times \mathbf{e}_c)^2 +$$

$$+ \varepsilon_\perp \varepsilon_\parallel g_e R_e^{-2} (1 - ikR_e) [\mathbf{R} \times \mathbf{e}_c] [\mathbf{R} \times (\mathbf{R} \times \mathbf{e}_c)] / (\mathbf{R} \times \mathbf{e}_c)^2, \tag{12}$$

$$- 4\pi\sqrt{\varepsilon_\perp}\; \boldsymbol{\epsilon}^{-1} \bullet \nabla \times \mathfrak{G}_m(\mathbf{R},\omega) = \varepsilon_\perp (g_e - g_o) (\mathbf{R} \bullet \mathbf{e}_c) [\mathbf{e}_c \times (\mathbf{R} \times \mathbf{e}_c)][\mathbf{R} \times \mathbf{e}_c] / (\mathbf{R} \times \mathbf{e}_c)^4 +$$

$$+ \varepsilon_\perp (g_e - g_o) (\mathbf{R} \bullet \mathbf{e}_c) [\mathbf{R} \times \mathbf{e}_c] [\mathbf{e}_c \times (\mathbf{R} \times \mathbf{e}_c)] / (\mathbf{R} \times \mathbf{e}_c)^4 -$$

$$- \varepsilon_\perp^2 g_o R_o^{-2} (1 - ikR_o) [\mathbf{R} \times \mathbf{e}_c][\mathbf{R} \times (\mathbf{R} \times \mathbf{e}_c)] / (\mathbf{R} \times \mathbf{e}_c)^2 +$$

$$+ \varepsilon_\perp \varepsilon_\parallel g_e R_e^{-2} (1 - ikR_e)[\mathbf{R} \times (\mathbf{R} \times \mathbf{e}_c)] [\mathbf{R} \times \mathbf{e}_c] / (\mathbf{R} \times \mathbf{e}_c)^2. \tag{13}$$

With reference to the expressions (9) - (13), the following notation holds:

$\mathbf{R} = \mathbf{r} - \mathbf{r}_s$; $\quad k = \omega\sqrt{(\varepsilon_o \mu_o)}$

$g_o = \exp[ikR_o]/R_o$; $\quad g_e = \exp[ikR_e]/R_e$

$R_o = \sqrt{[\varepsilon_\perp \mathbf{R} \bullet \mathbf{R}]}$, $\quad R_e = \sqrt{[\varepsilon_\parallel (\mathbf{R} \times \mathbf{e}_c)^2 + \varepsilon_\perp (\mathbf{R} \bullet \mathbf{e}_c)^2]}$.

It should be noted that the subscript 'o' refers to the *ordinary* waves, and 'e' pertains to the *extraordinary* waves (except for the time-harmonic electric Green's dyadic $\mathfrak{G}_e$), as is customary in crystal optics. The *ordinary* waves are the more familiar of the two because far from the source, an *extraordinary* wave still has an electric field component in the direction of the wave-normal, which the *ordinary* does not have. Furthermore, the phase velocity of an *ordinary* plane wave is independent of the direction of propagation, while that of an *extraordinary* plane wave depends on the propagation direction [6,7]. In other words, while the *ordinary* wave disregards the anisotropy of the unaxial medium, the *extraordinary* wave is sensitive to the difference between $\varepsilon_\perp$ and $\varepsilon_\parallel$.

Returning to the time-dependent case, the Green's dyadics $\mathfrak{G}_1$, etc. can be obtained by using temporal Fourier transforms as

$$\mathfrak{G}_1(\mathbf{R},t,t_s) = (\mu_o/2\pi) \int_{-\infty + i\Delta}^{\infty + i\Delta} d\omega (i\omega) \exp[- i\omega(t - t_s)]\, \mathfrak{G}_e(\mathbf{R},\omega), \tag{14}$$

$$\mathfrak{G}_2(\mathbf{R},t,t_s) = - (1/2\pi) \int_{-\infty + i\Delta}^{\infty + i\Delta} d\omega \exp [-i\omega(t - t_s)] [\, \boldsymbol{\epsilon}^{-1} \bullet \nabla \times \mathfrak{G}_m(\mathbf{R},\omega)], \tag{15}$$

$$\mathfrak{G}_3(\mathbf{R},t,t_s) = (1/2\pi) \int_{-\infty + i\Delta}^{\infty + i\Delta} d\omega \exp [-i\omega(t - t_s)] [\nabla \times \mathfrak{G}_e(\mathbf{R},\omega)], \tag{16}$$

$$\mathfrak{G}_4(\mathbf{R},t,t_s) = (\varepsilon_o/2\pi) \int_{-\infty + i\Delta}^{\infty + i\Delta} d\omega (i\omega) \exp[- i\omega(t - t_s)]\, \mathfrak{G}_m(\mathbf{R},\omega). \tag{17}$$

Physical considerations require that the solutions of (3) and (4) must exhibit three properties: (i) the medium is spatially invariant, (ii) the medium is temporally invariant, and (iii) the electromagnetic field must be causal. The first property has already been satisfied by the time-harmonic expressions (9) - (13), and the satisfaction of the second property is apparent in (14) - (17). The third property, i.e., causality implies that $\mathfrak{G}_1(\mathbf{R},t,t_s)$, etc., must be identically zero for $t < t_s$. This is obtained in (14) - (17) by invoking analytic continuation from real to complex ω by moving the integration path above the real axis in the complex ω plane; $\Delta > 0$ to ensure avoiding the pole singularities [8,9].

By substituting (9) - (13) in (14) - (17), and using the definitions [10]

$$\delta(t) = (1/\pi) \int_0^\infty d\omega \cos(\omega t), \tag{18}$$

$$u(t) = (1/2) + (1/\pi) \int_0^\infty d\omega \sin(\omega t) / \omega\,, \quad t \geq 0\,; \quad u(t) = 0, t < 0, \tag{19}$$

respectively, for the Dirac delta function $\delta(t)$ and the unit step function $u(t)$, it can be shown that

$$\begin{aligned} 4\pi(\varepsilon_\perp)^{1/2} (\mu_o)^{-1}\, \mathfrak{G}_1(\mathbf{R},\tau) = &- [\delta'(\tau_e) + (c/R_e)\, \delta(\tau_e) + (c/R_e)^2\, u(\tau_e)]\, (\varepsilon_\perp \varepsilon_\parallel / R_e)\, \mathfrak{e}^{-1} \\ &+ [\delta'(\tau_e) + 3(c/R_e)\, \delta(\tau_e) + 3(c/R_e)^2\, u(\tau_e)]\, (\varepsilon_\perp \varepsilon_\parallel / R_e)\, \mathfrak{A}(\mathbf{R}) \\ &+ [(\varepsilon_\parallel / R_e)\, \delta'(\tau_e) - (\varepsilon_\perp / R_o)\, \delta'(\tau_o)]\, \mathfrak{B}(\mathbf{R}) + [\delta(\tau_o) - \delta(\tau_e)]\, \mathfrak{C}(\mathbf{R}), \end{aligned} \tag{20}$$

$$\begin{aligned} 4\pi(\varepsilon_\perp)^{-1/2}\, \mathfrak{G}_2(\mathbf{R},\tau) = &[\delta(\tau_e)/R_e - \delta(\tau_o)/R_o]\, \mathfrak{D}(\mathbf{R}) - \\ &\varepsilon_\perp R_o^{-3} [\delta(\tau_o) + (R_o/c)\, \delta'(\tau_o)]\, \mathfrak{E}(\mathbf{R}) + \\ &\varepsilon_\parallel R_e^{-3} [\delta(\tau_e) + (R_e/c)\, \delta'(\tau_e)]\, \mathfrak{F}(\mathbf{R}), \end{aligned} \tag{21}$$

and

$$\begin{aligned} 4\pi(\varepsilon_\perp)^{-1/2}\, \mathfrak{G}_3(\mathbf{R},\tau) = &[\delta(\tau_o)/R_e - \delta(\tau_o)/R_o]\, \mathfrak{D}(\mathbf{R}) - \\ &\varepsilon_\perp R_o^{-3} [\delta(\tau_o) + (R_o/c)\, \delta'(\tau_o)]\, \mathfrak{F}(\mathbf{R}) + \\ &\varepsilon_\parallel R_e^{-3} [\delta(\tau_e) + (R_e/c)\, \delta'(\tau_e)]\, \mathfrak{E}(\mathbf{R}), \end{aligned} \tag{22}$$

$$4\pi\,(\epsilon_\perp)^{-1/2}(\epsilon_o)^{-1}\,\mathfrak{G}_4(\mathbf{R},\tau) = -\,[\delta'(\tau_o) + (c/R_o)\,\delta(\tau_o) + (c/R_o)^2\,u(\tau_o)]\,(\epsilon_\perp/R_0)\,\mathfrak{I}$$

$$+\,[\,\delta'(\tau_o) + 3(c/R_o)\,\delta(\tau_o) + 3(c/R_o)^2\,u(\tau_o)]\,(\epsilon_\perp/R_o)\,\mathbf{e}_R\mathbf{e}_R$$

$$-\,[(\epsilon_{||}/R_e)\,\delta'(\tau_e) - (\epsilon_\perp/R_o)\,\delta'(\tau_o)]\,\mathfrak{B}(\mathbf{R}) - [\delta(\tau_o) - \delta(\tau_e)]\,\mathfrak{C}(\mathbf{R}). \quad (23)$$

In the expressions (20) - (23), $\delta'(\zeta) = \{d/d\zeta\}\delta(\zeta)$ is the doublet impulse function [10], and for the sake of convenience, the following notation has been used:

$\tau = t - t_s$; $\qquad c = (\mu_o\epsilon_o)^{-1/2}$,

$\tau_o = t - t_s - R_o/c$; $\qquad \tau_e = t - t_s - R_e/c$,

$\mathbf{e}_R = \mathbf{R}/R$; $\qquad \mathfrak{A}(\mathbf{R}) = (\epsilon_\perp\epsilon_{||}/R_e^2)[\boldsymbol{\epsilon}^{-1}\bullet\mathbf{R}][\boldsymbol{\epsilon}^{-1}\bullet\mathbf{R}]$,

$\mathfrak{B}(\mathbf{R}) = [\mathbf{R}\times\mathbf{e}_c][\mathbf{R}\times\mathbf{e}_c]\,/\,(\mathbf{R}\times\mathbf{e}_c)^2$; $\qquad \mathfrak{C}(\mathbf{R}) = c\,[\mathfrak{I} - \mathbf{e}_c\mathbf{e}_c - 2\mathfrak{B}(\mathbf{R})]\,/\,(\mathbf{R}\times\mathbf{e}_c)^2$,

$\mathfrak{D}(\mathbf{R}) = (\mathbf{R}\bullet\mathbf{e}_c)\left\{[\mathbf{R}\times\mathbf{e}_c][\mathbf{e}_c\times(\mathbf{R}\times\mathbf{e}_c)] + [\mathbf{e}_c\times(\mathbf{R}\times\mathbf{e}_c)]\,[\mathbf{R}\times\mathbf{e}_c)]\right\}/\,(\mathbf{R}\times\mathbf{e}_c)^4$,

$\mathfrak{E}(\mathbf{R}) = [\mathbf{R}\times\mathbf{e}_c][\mathbf{R}\times(\mathbf{R}\times\mathbf{e}_c)]\,/\,(\mathbf{R}\times\mathbf{e}_c)^2$,

$\mathfrak{F}(\mathbf{R}) = [\mathbf{R}\times(\mathbf{R}\times\mathbf{e}_c)][\mathbf{R}\times\mathbf{e}_c]\,/\,(\mathbf{R}\times\mathbf{e}_c)^2$.

It should be noted that τ_o is the retarded time for the *ordinary* field, and τ_e is for the *extraordinary* field. Furthermore, $\delta(\tau_o)$, $\delta(\tau_e)$, $\delta'(\tau_o)$ and $\delta'(\tau_e)$ are switching functions that establish the "wave head" in a given direction. For the sake of illustration, let the sources $\mathbf{J}(\mathbf{r},t)$ and $\mathbf{K}(\mathbf{r},t)$ be of the form $\delta(\mathbf{r})\,\delta(t)$. Then, the *ordinary* wave head moves with a velocity

$$v_o = c\,/\,\sqrt{(\epsilon_\perp)}$$

isotropically; whereas the *extraordinary* wave head has a velocity

$$v_e = c\,/\,\sqrt{\,[\epsilon_{||}\sin^2\theta + \epsilon_\perp\cos^2\theta]}$$

which is dependent upon the azimuthal angle $\theta = \arctan\{|\mathbf{r}\times\mathbf{e}_c|/(\mathbf{r}\bullet\mathbf{e}_c)\}$. Thus, in the direction $\mathbf{e}_r$, the point $t = r_o/c$ (resp. $t = r_e/c$) is the *overall* wave head when $\epsilon_{||} > \epsilon_\perp$ (resp. $\epsilon_{||} < \epsilon_\perp$). Shown in Fig. 1 are calculated values of v_e/v_o as functions of θ for several values of the material anisotropy parameter $\epsilon_{||}/\epsilon_\perp$.

Next, electric sources $\mathbf{J}(\mathbf{r},t) = \mathbf{e}_c\,J(\mathbf{r},t)$, where J is at least once differentiable, give rise only to the extraordinary fields. Likewise, magnetic sources $\mathbf{K}(\mathbf{r},t) = \mathbf{e}_c\,K(\mathbf{r},t)$ radiate only the ordinary fields. This can be deduced by noting that $\mathfrak{B}(\mathbf{R})\bullet\mathbf{e}_c$, $\mathfrak{C}(\mathbf{R})\bullet\mathbf{e}_c$, $\mathfrak{D}(\mathbf{R})\bullet\mathbf{e}_c$ and $\mathfrak{F}(\mathbf{R})\bullet\mathbf{e}_c$ are all identically zero.

The time-dependent Green's dyadics derived here reflect the fact that the uniaxial medium is spatially reciprocal and possess spatial symmetry, as per the transpose relations,

$$\mathfrak{G}_1(\mathbf{R},\tau) = [\mathfrak{G}_1(\mathbf{R},\tau)]^{tr}; \qquad \mathfrak{G}_4(\mathbf{R},\tau) = [\mathfrak{G}_4(\mathbf{R},\tau)]^{tr}, \qquad (24a,\ b)$$

$$\mathfrak{G}_2(\mathbf{R},\tau) = [\mathfrak{G}_3(\mathbf{R},\tau)]^{tr}; \qquad \mathfrak{G}_3(\mathbf{R},\tau) = [\mathfrak{G}_2(\mathbf{R},\tau)]^{tr}, \qquad (24c,\ d)$$

and the symmetry relations,

$$\mathfrak{G}_1(\mathbf{R},\tau) = \mathfrak{G}_1(-\mathbf{R},\tau); \qquad \mathfrak{G}_4(\mathbf{R},\tau) = \mathfrak{G}_4(-\mathbf{R},\tau), \qquad (25a,b)$$

$$\mathfrak{G}_2(\mathbf{R},\tau) = -\,\mathfrak{G}_2(-\mathbf{R},\tau); \qquad \mathfrak{G}_3(\mathbf{R},\tau) = -\,\mathfrak{G}_3(-\mathbf{R},\tau). \qquad (25c,d)$$

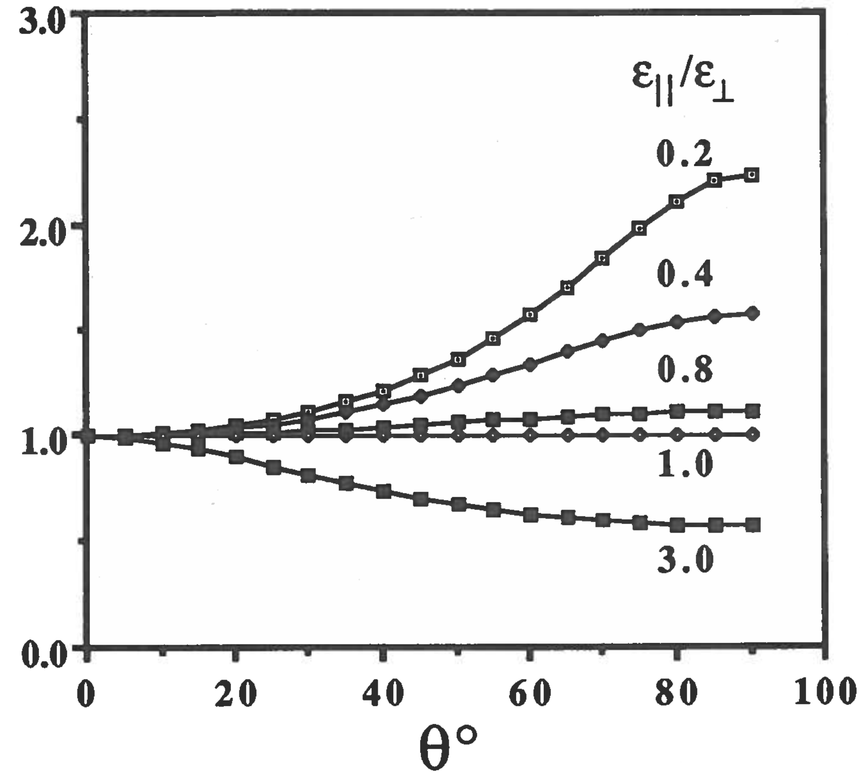

Figure 1 Computed ratio v_e/v_o of the wave head velocities as functions of the angle $\theta = \arctan\{|\mathbf{r}\times\mathbf{e}_c|/(\mathbf{r}\cdot\mathbf{e}_c)\}$ for several values of the material anisotropy parameter $\varepsilon_{||}/\varepsilon_{\perp}$.

3. THE INITIAL VALUE PROBLEM

Very often, the spatial distribution of the electromagnetic field at time t = 0 is known, and it is desired to compute the electric and the magnetic fields at later times, in the absence of other sources. Typically, this is of interest in EMP propagation problems where the initial field is confined to some finite volume V_q, and the calculations of $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{H}(\mathbf{r},t)$ are required for $t > 0$ and $\mathbf{r} \notin V_q$. For that purpose, consider (3) and (4) in which the source terms have been nulled. Furthermore, let

$$\mathbf{E}_0(\mathbf{r}) = \mathbf{E}(\mathbf{r},t)\,|_{t=0+}\,; \qquad \mathbf{E}_1(\mathbf{r}) = \{\partial/\partial t\}\,\mathbf{E}(\mathbf{r},t)\,|_{t=0+}\,, \tag{26a,b}$$

$$\mathbf{H}_0(\mathbf{r}) = \mathbf{H}(\mathbf{r},t)\,|_{t=0+}\,; \qquad \mathbf{H}_1(\mathbf{r}) = \{\partial/\partial t\}\,\mathbf{H}(\mathbf{r},t)\,|_{t=0+}\,, \tag{27a,b}$$

The temporal Fourier transforms of (3) and (4) should next be taken, bearing in mind that the solution of the initial value problem should not violate causality; as is apparent from the discussion following (17), the Laplace transforms may be taken alternatively. In either case, it can be shown that

$$\mathbf{E}(\mathbf{r},t) = \mu_o\varepsilon_o \iiint_{V_q} d^3\mathbf{r}_q \left\{ (1/2\pi) \int_{-\infty+i\Delta}^{\infty+i\Delta} d\omega (i\omega) \exp[-i\omega t]\, \mathfrak{B}_e(\mathbf{R},\omega)\cdot\underline{\epsilon}\cdot\mathbf{E}_0(\mathbf{r}_q) \right\}$$

$$+ \mu_o\varepsilon_o \iiint_{V_q} d^3\mathbf{r}_q \left\{ (1/2\pi) \int_{-\infty+i\Delta}^{\infty+i\Delta} d\omega \exp[-i\omega t]\, \mathfrak{B}_e(\mathbf{R},\omega)\cdot\underline{\epsilon}\cdot\mathbf{E}_1(\mathbf{r}_q) \right\}, \tag{28}$$

$$\mathbf{H}(\mathbf{r},t) = \mu_o\varepsilon_o \iiint_{V_q} d^3r_q \left\{ (1/2\pi) \int_{-\infty + i\Delta}^{\infty + i\Delta} d\omega (i\omega) \exp[-i\omega t]\, \mathfrak{G}_m(\mathbf{R}, \omega) \bullet \mathbf{H}_0(\mathbf{r}_q) \right\}$$

$$+ \mu_o\varepsilon_o \iiint_{V_q} d^3r_q \left\{ (1/2\pi) \int_{-\infty + i\Delta}^{\infty + i\Delta} d\omega \exp[-i\omega t]\, \mathfrak{G}_m(\mathbf{R},\omega) \bullet \mathbf{H}_1(\mathbf{r}_q) \right\}, \tag{29}$$

with $\mathbf{R} = \mathbf{r} - \mathbf{r}_q$. On noting the properties of the integral transformations used, and in view of (14) and (17), the solution of the initial value problem works out to be

$$\mathbf{E}(\mathbf{r},t > 0) = \varepsilon_o \iiint_{V_q} d^3r_q \left\{ \mathfrak{G}_1(\mathbf{R},t) \bullet \mathfrak{e} \bullet \mathbf{E}_0(\mathbf{r}_q) - \mathfrak{G}_5(\mathbf{R},t) \bullet \mathfrak{e} \bullet \mathbf{E}_1(\mathbf{r}_q) \right\}, \tag{30}$$

$$\mathbf{H}(\mathbf{r},t > 0) = \mu_o \iiint_{V_q} d^3r_q \left\{ \mathfrak{G}_4(\mathbf{R},t) \bullet \mathbf{H}_0(\mathbf{r}_q) - \mathfrak{G}_6(\mathbf{R},t) \bullet \mathbf{H}_1(\mathbf{r}_q) \right\}, \tag{31}$$

where

$$\mathfrak{G}_5(\mathbf{R},t) = \int_0^t dt\, \mathfrak{G}_1(\mathbf{R},t); \qquad \mathfrak{G}_6(\mathbf{R},t) = \int_0^t dt\, \mathfrak{G}_4(\mathbf{R},t). \tag{32a,b}$$

Explicit expressions for $\mathfrak{G}_5$ and $\mathfrak{G}_6$, respectively, are as follows:

$$\begin{aligned} 4\pi(\varepsilon_\perp)^{1/2} (\mu_o)^{-1}\, \mathfrak{G}_5(\mathbf{R},t) = &- [\delta(\tau_e) + (c/R_e)\, u(\tau_e) + (c/R_e)^2\, \tau_e\, u(\tau_e)]\, (\varepsilon_\perp\varepsilon_\parallel / R_e)\, \mathfrak{e}^{-1} \\ &+ [\delta(\tau_e) + 3(c/R_e)\, u(\tau_e) + 3(c/R_e)^2\, \tau_e\, u(\tau_e)]\, (\varepsilon_\perp\varepsilon_\parallel / R_e)\, \mathfrak{A}(\mathbf{R}) \\ &+ [\, (\varepsilon_\parallel / R_e)\, \delta(\tau_e) - (\varepsilon_\perp / R_o)\, \delta(\tau_o)]\, \mathfrak{B}(\mathbf{R}) + [u(\tau_o) - u(\tau_e)]\, \mathfrak{C}(\mathbf{R}), \end{aligned} \tag{33}$$

$$\begin{aligned} 4\pi(\varepsilon_\perp)^{-1/2} (\varepsilon_o)^{-1}\, \mathfrak{G}_6(\mathbf{R},t) = &-[\delta(\tau_o) + (c/R_o)\, u(\tau_o) + (c/R_o)^2 \tau_o\, u(\tau_o)]\, (\varepsilon_\perp / R_o)\, \mathfrak{I} \\ &+ [\, \delta(\tau_o) + 3(c/R_o)\, u(\tau_o) + 3(c/R_o)^2 \tau_o\, u(\tau_o)]\, (\varepsilon_\perp/R_o)\, \mathbf{e}_R\mathbf{e}_R \\ &- [(\varepsilon_\parallel / R_e)\, \delta(\tau_e) - (\varepsilon_\perp / R_o)\, \delta(\tau_o)]\, \mathfrak{B}(\mathbf{R}) - [u(\tau_o) - u(\tau_e)]\, \mathfrak{C}(\mathbf{R}), \end{aligned} \tag{34}$$

in which $\tau_e = t - R_e/c$ and $\tau_o = t - R_o/c$. Using (2a,b), (26a,b) and (27a,b), alternative forms of (30) and (31) can also be obtained as, respectively,

$$\mathbf{E}(\mathbf{r},t > 0) = \iiint_{V_q} d^3r_q \left\{ \mathfrak{G}_1(\mathbf{R},t) \bullet \varepsilon_o \mathfrak{e} \bullet \mathbf{E}_0(\mathbf{r}_q) - \mathfrak{G}_5(\mathbf{R},t) \bullet \nabla_q \times \mathbf{H}_0(\mathbf{r}_q) \right\}, \tag{35}$$

$$\mathbf{H}(\mathbf{r},t > 0) = \iiint_{V_q} d^3r_q \left\{ \mathfrak{G}_4(\mathbf{R},t) \bullet \mu_o \mathbf{H}_0(\mathbf{r}_q) + \mathfrak{G}_6(\mathbf{R},t) \bullet \nabla_q \times \mathbf{E}_0(\mathbf{r}_q) \right\}. \tag{36}$$

4. RADIATION FROM CONCENTRATED SOURCES

With the derivation of the time-dependent Green's functions, it becomes possible to compute the fields radiated by source current densities using the expressions (5) and (6). In this section, we will consider the radiation characteristics of several elementary sources which are spatially concentrated at $\mathbf{r}_s = \mathbf{0}$, but have time-dependent excitations. In the sequel, $\tau = t$, $\tau_e = t - r_e/c$ and $\tau_o = t - r_o/c$, while $\mathbf{R} = \mathbf{r}$.

(i) *Electric dipole flashing on and off at t = 0.* An electric dipole flashing on and off at t = 0 gives rise to the electric source

$$\mathbf{J}(\mathbf{r}_s,t_s) = \mathbf{e}_p\, \delta(\mathbf{r}_s)\, \delta'(t_s), \tag{37}$$

where $\mathbf{e}_p$ represents the orientation of the electric dipole. The radiated fields can be obtained as

$$\begin{aligned} 4\pi(\varepsilon_\perp)^{1/2}(\mu_o)^{-1}\, \mathbf{E}(\mathbf{r},t) = \Big\{ & [-\delta''(\tau_e) - (c/r_e)\,\delta'(\tau_e) - (c/r_e)^2\,\delta(\tau_e)]\,(\varepsilon_\perp\varepsilon_\parallel / r_e)\,\boldsymbol{\epsilon}^{-1} \\ & + [\delta''(\tau_e) + 3(c/r_e)\,\delta'(\tau_e) + 3(c/r_e)^2\,\delta(\tau_e)]\,(\varepsilon_\perp\varepsilon_\parallel / r_e)\,\mathfrak{A}(\mathbf{r}) \\ & + [(\varepsilon_\parallel / r_e)\,\delta''(\tau_e) - (\varepsilon_\perp / r_o)\,\delta''(\tau_o)]\,\mathfrak{B}(\mathbf{r}) \\ & + [\delta'(\tau_o) - \delta'(\tau_e)]\,\mathfrak{C}(\mathbf{r}) \Big\} \bullet \mathbf{e}_p, \end{aligned} \tag{38a}$$

$$\begin{aligned} 4\pi(\varepsilon_\perp)^{-1/2}\, \mathbf{H}(\mathbf{r},t) = \Big\{ & [\delta'(\tau_e)/r_e - \delta'(\tau_o)/r_o]\,\mathfrak{D}(\mathbf{R}) - \\ & \varepsilon_\perp r_o^{-3}[\delta'(\tau_o) + (r_o/c)\,\delta''(\tau_o)]\,\mathfrak{F}(\mathbf{r}) + \\ & \varepsilon_\parallel r_e^{-3}[\delta'(\tau_e) + (r_e/c)\,\delta''(\tau_e)]\,\mathfrak{E}(\mathbf{r}) \Big\} \bullet \mathbf{e}_p, \end{aligned} \tag{38b}$$

in which $\delta''(\zeta) = \{d/d\zeta\}\delta'(\zeta)$ is the triplet impulse function [10]. Thus, (38a,b) constitute the response of the uniaxial medium to an electric impulse, and it should be mentioned that $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{H}(\mathbf{r},t)$ are identically zero at large times t such that $t > r_e/c$ and $t > r_o/c$.

(ii) *Magnetic dipole flashing on and off at t = 0.* A magnetic dipole flashing on and off at t = 0 gives rise to the magnetic source

$$\mathbf{K}(\mathbf{r}_s,t_s) = \mathbf{e}_m\, \delta(\mathbf{r}_s)\, \delta'(t_s), \tag{39}$$

where $\mathbf{e}_m$ represents the orientation of the magnetic dipole. The radiated fields can be obtained as

$$\begin{aligned} 4\pi(\varepsilon_\perp)^{-1/2}\, \mathbf{E}(\mathbf{r},t) = \Big\{ & [\delta'(\tau_e)/r_e - \delta'(\tau_o)/r_o]\,\mathfrak{D}(\mathbf{r}) - \varepsilon_\perp r_o^{-3}\,[\delta'(\tau_o) + (r_o/c)\,\delta''(\tau_o)]\,\mathfrak{E}(\mathbf{r}) + \\ & \varepsilon_\parallel r_e^{-3}[\delta'(\tau_e) + (r_e/c)\,\delta''(\tau_e)]\,\mathfrak{F}(\mathbf{r}) \Big\} \bullet \mathbf{e}_m, \end{aligned} \tag{40a}$$

$$\begin{aligned} 4\pi(\varepsilon_\perp)^{-1/2}(\varepsilon_o)^{-1}\, \mathbf{H}(\mathbf{r},t) = \Big\{ & [-\delta''(\tau_o) - (c/r_o)\,\delta'(\tau_o) - (c/r_o)^2\,\delta(\tau_o)]\,(\varepsilon_\perp/r_o)\mathfrak{I} \\ & + [\delta''(\tau_o) + 3(c/r_o)\,\delta'(\tau_o) + 3(c/r_o)^2\,\delta(\tau_o)]\,(\varepsilon_\perp/r_o)\,\mathbf{r}\mathbf{r}/r^2 \\ & + [(\varepsilon_\parallel/r_o)\,\delta''(\tau_o) - (\varepsilon_\perp/r_e)\,\delta''(\tau_e)]\,\mathfrak{B}(\mathbf{r}) \end{aligned}$$

$$+ [\delta'(\tau_e) - \delta'(\tau_o)]\, \mathfrak{C}(\mathbf{r}) \Big\} \bullet \mathbf{e}_m. \tag{40b}$$

Equations (40a,b) constitute the response of the uniaxial medium to a magnetic impulse, and it should be noted again that $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{H}(\mathbf{r},t)$ are identically zero at large times t such that $t > r_e/c$ and $t > r_o/c$.

(iii) *Electric dipole turning on at t = 0.* An electric dipole turning on at $t = 0$ gives rise to the electric source

$$\mathbf{J}(\mathbf{r}_s,t_s) = \mathbf{e}_p\, \delta(\mathbf{r}_s)\, \delta(t_s), \tag{41}$$

where $\mathbf{e}_p$ represents the orientation of the electric dipole. The radiated fields can be obtained as

$$4\pi(\varepsilon_\perp)^{1/2}\,(\mu_o)^{-1}\,\mathbf{E}(\mathbf{r},t) = \Big\{ [-\delta'(\tau_e) - (c/r_e)\,\delta(\tau_e) - (c/r_e)^2\, u(\tau_e)]\,(\varepsilon_\perp\varepsilon_\parallel / r_e)\, \boldsymbol{\epsilon}^{-1}$$

$$+ [\delta'(\tau_e) + 3(c/r_e)\,\delta(\tau_e) + 3(c/r_e)^2\, u(\tau_e)]\,(\varepsilon_\perp\varepsilon_\parallel / r_e)\, \mathfrak{A}(\mathbf{r})$$

$$+ [(\varepsilon_\parallel/r_e)\,\delta'(\tau_e) - (\varepsilon_\perp/r_o)\,\delta'(\tau_o)]\, \mathfrak{B}(\mathbf{r})$$

$$+ [\delta(\tau_o) - \delta(\tau_e)]\, \mathfrak{C}(\mathbf{r}) \Big\} \bullet \mathbf{e}_p, \tag{42a}$$

$$4\pi(\varepsilon_\perp)^{-1/2}\,\mathbf{H}(\mathbf{r},t) = \Big\{ [\delta(\tau_e)/r_e - \delta(\tau_o)/r_o]\, \mathfrak{D}(\mathbf{R}) - \varepsilon_\perp r_o^{-3} [\delta(\tau_o) + (r_o/c)\,\delta'(\tau_o)]\, \mathfrak{F}(\mathbf{r})$$

$$+ \varepsilon_\parallel r_e^{-3} [\delta(\tau_e) + (r_e/c)\,\delta'(\tau_e)]\, \mathfrak{E}(\mathbf{r}) \Big\} \bullet \mathbf{e}_p. \tag{42b}$$

Thus, (42a,b) constitute the response of the uniaxial medium to an electric unit step. At large times t such that $t > r_e/c$ and $t > r_o/c$, the radiated fields are given by

$$4\pi\,(\varepsilon_\perp)^{1/2}\,(\mu_o)^{-1}\,\mathbf{E}(\mathbf{r},t) = (c/r_e)^2\,(\varepsilon_\perp\varepsilon_\parallel/r_e)\,\Big\{ 3(\varepsilon_\perp\varepsilon_\parallel/r_e^2)\,[\boldsymbol{\epsilon}^{-1}\bullet\mathbf{r}]\,\mathbf{r} - \mathfrak{I} \Big\}\,(\boldsymbol{\epsilon}^{-1}\bullet\mathbf{e}_p), \tag{43a}$$

$$\mathbf{H}(\mathbf{r},t) = 0, \tag{43b}$$

which are nothing but the fields radiated by a static electric dipole in the uniaxial dielectric medium. It should be noted that by setting $\varepsilon_\perp = \varepsilon_\parallel$ in (43a,b) the field due to a static electric dipole in an isotropic medium can be recovered [11].

(iv) *Magnetic dipole turning on at t = 0.* A magnetic dipole turning on at $t = 0$ gives rise to the magnetic source

$$\mathbf{K}(\mathbf{r}_s,t_s) = \mathbf{e}_m\, \delta(\mathbf{r}_s)\, \delta(t_s), \tag{44}$$

where $\mathbf{e}_m$ represents the orientation of the magnetic dipole. The radiated fields can be derived as

$$4\pi\,(\varepsilon_\perp)^{-1/2}\,\mathbf{E}(\mathbf{r},t) = \Big\{ [\delta(\tau_e)/r_e - \delta(\tau_o)/r_o]\, \mathfrak{D}(\mathbf{r}) - \varepsilon_\perp r_o^{-3} [\delta(\tau_o) + (r_o/c)\,\delta'(\tau_o)]\, \mathfrak{E}(\mathbf{r})$$

$$+ \varepsilon_\parallel r_e^{-3}\, [\delta(\tau_e) + (r_e/c)\,\delta'(\tau_e)]\, \mathfrak{F}(\mathbf{r}) \Big\} \bullet \mathbf{e}_m, \tag{45a}$$

$$4\pi(\varepsilon_\perp)^{-1/2}\,(\varepsilon_o)^{-1}\,\mathbf{H}(\mathbf{r},t) = \Big\{ [-\delta'(\tau_o) - (c/r_o)\,\delta(\tau_o) - (c/r_o)^2\, u(\tau_o)]\,(\varepsilon_\perp/r_o)\, \mathfrak{I}$$

$$+ [\delta'(\tau_o) + 3(c/r_o)\,\delta(\tau_o) + 3(c/r_o)^2\, u(\tau_o)]\,\,(\varepsilon_\perp / r_o)\, \mathbf{r}\mathbf{r}/r^2$$

$$+ [(\varepsilon_\parallel/r_o)\,\delta'(\tau_o) - (\varepsilon_\perp/r_e)\,\delta'(\tau_e)]\, \mathfrak{B}(\mathbf{r})$$

$$+ [\delta(\tau_e) - \delta(\tau_o)]\, \mathfrak{C}(r) \Big\} \cdot e_m. \tag{45b}$$

Equations (45a,b) constitute the response of the uniaxial medium to a magnetic unit step. At large times t such that $t > r_e/c$ and $t > r_o/c$, the radiated fields are given by

$$E(r, t) = \mathbf{0} \tag{46a}$$

$$4\pi (\varepsilon_\perp)^{-1/2} (\varepsilon_o)^{-1} H(r,t) = (c/r_o)^2 (\varepsilon_\perp/r_o) [\, 3rr/r^2 - \mathfrak{I}] \cdot e_m, \tag{46b}$$

which are the fields due to a static magnetic dipole in an *isotropic* medium having a relative dielectric constant $\varepsilon_\perp$ [11]. It should also be mentioned that expressions (45a,b), with $\varepsilon_\perp = \varepsilon_\parallel$, are the same as can be derived using Wait's analysis [12] for transient magnetic dipoles in isotropic media.

(v) *Suddenly initiated time-harmonic electric dipole.* The sudden initiation of an electric dipole by a time-harmonic signal of constant frequency ω_m can be described by the electric current density

$$J(r_s,t_s) = e_p\, \delta(r_s) \cos(\omega_m t_s)\, u(t_s). \tag{47}$$

Substitution of (47) in (5) and (6) yields the radiated fields as

$$\begin{aligned}4\pi(\varepsilon_\perp)^{1/2} (\mu_0)^{-1} E(r,t) = {}& \delta(\tau_e)\, (\varepsilon_\parallel/r_e) \Big\{ \varepsilon_\perp \mathfrak{A}(r) - \varepsilon_\perp \mathfrak{t}^{-1} + \mathfrak{B}(r) \Big\} \cdot e_p - \delta(\tau_o)(\varepsilon_\perp/r_o)\, \mathfrak{B}(r) \cdot e_p \\ & + u(\tau_e) \Big\{ \Big([c^2 r_e^{-2} - \omega_m] \sin(\omega_m \tau_e) + c r_e^{-1} \cos(\omega_m \tau_e) \Big) (\varepsilon_\perp \varepsilon_\parallel / r_e)\, \mathfrak{t}^{-1} \\ & + \Big([3c^2 r_e^{-2} - \omega_m] \sin(\omega_m \tau_e) + 3c\, r_e^{-1} \cos(\omega_m \tau_e) \Big)\, \mathfrak{A}(r) \\ & - \omega_m \varepsilon_\parallel\, r_e^{-1} \sin(\omega_m \tau_e)\, \mathfrak{B}(r) - \cos(\omega_m \tau_e)\, \mathfrak{C}(r) \Big\} \cdot e_p \\ & + u(\tau_o) \Big\{ \omega_m\, \varepsilon_\perp\, r_o^{-1} \sin(\omega_m \tau_o)\, \mathfrak{B}(r) + \cos(\omega_m \tau_o)\, \mathfrak{C}(r) \Big\} \cdot e_p,\end{aligned} \tag{48a}$$

$$\begin{aligned}4\pi(\varepsilon_\perp)^{-1/2} H(r,t) = {}& \Big\{ \delta(\tau_e)\, (\varepsilon_\parallel / c r_e^2)\, \mathfrak{E}(r) - \delta(\tau_o)\, (\varepsilon_\perp / c r_o^2)\, \mathfrak{F}(r) \Big\} \cdot e_p \\ & + u(\tau_e) \Big\{ \cos(\omega_m \tau_e) \Big(\mathfrak{D}(r) / r_e + \varepsilon_\parallel\, r_e^{-3}\, \mathfrak{F}(r) \Big) - \omega_m \varepsilon_\parallel\, r_e^{-2} c^{-1} \sin(\omega_m \tau_e)\, \mathfrak{F}(r) \Big\} \cdot e_p \\ & - u(\tau_o) \Big\{ \cos(\omega_m \tau_o) \Big(\mathfrak{D}(r) / r_o + \varepsilon_\perp\, r_o^{-3}\, \mathfrak{E}(r) \Big) - \omega_m \varepsilon_\perp\, r_o^{-2} c^{-1} \sin(\omega_m \tau_o)\, \mathfrak{E}(r) \Big\} \cdot e_p.\end{aligned} \tag{48b}$$

In order to obtain the large-time behavior, terms containing the Dirac deltas in (48a,b) should be nulled, while the unit step functions should be replaced by unity.

(vi) *Suddenly initiated time-harmonic magnetic dipole.* The sudden initiation of a magnetic dipole by a time-harmonic signal of constant frequency ω_m can be described by the magnetic current density

$$K(r_s,t_s) = e_m\, \delta(r_s) \cos(\omega_m t_s)\, u(t_s); \tag{49}$$

and the substitution of (38) in (5) and (6) yields the radiated fields as

$$\begin{aligned}4\pi(\varepsilon_\perp)^{-1/2} E(r,t) = {}& \Big\{ \delta(\tau_e)\, (\varepsilon_\parallel / c r_e^2)\, \mathfrak{E}(r) - \delta(\tau_o)\, (\varepsilon_\perp / c r_o^2)\, \mathfrak{F}(r) \Big\} \cdot e_m \\ & + u(\tau_e) \Big\{ \cos(\omega_m \tau_e) \Big(\mathfrak{D}(r) / r_e + \varepsilon_\parallel\, r_e^{-3}\, \mathfrak{E}(r) \Big) - \omega_m \varepsilon_\parallel\, r_e^{-2} c^{-1} \sin(\omega_m \tau_e)\, \mathfrak{E}(r) \Big\} \cdot e_m \\ & - u(\tau_o) \Big\{ \cos(\omega_m \tau_o) \Big(\mathfrak{D}(r) / r_o + \varepsilon_\perp\, r_o^{-3}\, \mathfrak{F}(r) \Big) - \omega_m \varepsilon_\perp\, r_o^{-2} c^{-1} \sin(\omega_m \tau_o)\, \mathfrak{F}(r) \Big\} \cdot e_m.\end{aligned} \tag{50a}$$

$$\begin{aligned}4\pi(\epsilon_\perp)^{-1/2}(\epsilon_o)^{-1}\,\mathbf{H}(\mathbf{r},t) &= \delta(\tau_o)\,(\epsilon_\perp/r_o)\left\{r^{-2}\mathbf{rr}-\mathfrak{I}+\mathfrak{B}(\mathbf{r})\right\}\bullet\mathbf{e}_m\\
&\quad-\delta(\tau_e)\,(\epsilon_\parallel/r_e)\,\mathfrak{B}(\mathbf{r})\bullet\mathbf{e}_m\\
&\quad+u(\tau_o)\left\{\cos(\omega_m\tau_o)\left(\epsilon_\perp c r_o^{-2}[3\,r^{-2}\mathbf{rr}-\mathfrak{I}]-\mathfrak{C}(\mathbf{r})\right)\right.\\
&\quad\left.+\sin(\omega_m\tau_o)\left(\omega_m\epsilon_\perp r_o^{-1}[\mathfrak{I}-r^{-2}\mathbf{rr}-\mathfrak{B}(\mathbf{r})]+c(\omega_m r_o)^{-1}[3r^{-2}\mathbf{rr}-\mathfrak{I}]\right)\right\}\bullet\mathbf{e}_m\\
&\quad+u(\tau_e)\left\{\omega_m\epsilon_\parallel r_e^{-1}\sin(\omega_m\tau_e)\,\mathfrak{B}(\mathbf{r})+\cos(\omega_m\tau_e)\,\mathfrak{C}(\mathbf{r})\right\}\bullet\mathbf{e}_m. \qquad (50b)\end{aligned}$$

As in the case of the electric dipole of case (v) above, the large-time fields can be obtained by ignoring the terms containing the Dirac deltas in (50a,b), while the unit step functions are replaced by unity.

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