

REGARDING THE SOURCES OF RADIATION FIELDS IN AN ISOTROPIC CHIRAL MEDIUM

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ABSTRACT

The sources of left- and right- circularly polarized (LCP and RCP) waves in an isotropic chiral medium ($\mathbf{D} = \epsilon \mathbf{E} + \beta \nabla \times \mathbf{E}$, $\mathbf{B} = \mu \mathbf{H} + \beta \nabla \times \mathbf{H}$) have been explored here, as well as the radial and the transverse components of the electromagnetic fields examined. Auxilliary functions, Q_1 and Q_2 , for the LCP and the RCP fields, respectively are defined, and their eigenfunctions set up. The radiated fields of some simple sources are then investigated.

1. PRELIMINARIES

In 1948, Tellegen [1] proposed the most general constitutive equations for a linear, anisotropic medium, given as

$$\mathbf{D} = \underline{\epsilon} \cdot \mathbf{E} + \underline{\zeta} \cdot \mathbf{H}; \quad \mathbf{B} = \underline{\xi} \cdot \mathbf{E} + \underline{\mu} \cdot \mathbf{H}, \quad (1a,b)$$

in which $\underline{\epsilon}$, $\underline{\mu}$, $\underline{\zeta}$ and $\underline{\xi}$ are full, complex tensors, in general. The potency of these constitutive equations [2] is such that they encompass gyroelectric, gyromagnetic and gyro-electric-magnetic media, as well as optically active media, and a uniform treatment of all linear media became possible [3-7] for some electromagnetic problems. Whereas gyro-electric and gyro-magnetic media have been heavily investigated in the past due to their relevance to magnetoplasmas [8,9] and some classes of artificial dielectrics [10], isotropic optically active (chiral) media have not been paid much attention to by electromagnetic field theorists. This is largely because natural optically active substances have fallen in the province of physical chemists. With modern advances in polymer science [11-13], however, there is reason to believe that artificial chiral dielectrics, active at the mm-wave frequencies, may become feasible.

In order to describe the electromagnetic properties of an *isotropic* chiral medium, it is more convenient to use the constitutive relations [14]

$$\mathbf{D} = \epsilon [\mathbf{E} + \beta \nabla \times \mathbf{E}]; \quad \mathbf{B} = \mu [\mathbf{H} + \beta \nabla \times \mathbf{H}], \quad (2a,b)$$

than the Tellegen equations (1) because the curl is not a vector under the mirror inversion of the co-ordinate system; the pseudoscalar chirality parameter $\beta > 0$ (resp. < 0) for a right- (resp. left) handed medium, while ϵ and μ , respectively, are its permittivity and permeability. These relations are symmetric under time-reversality [15] and duality transformations [16]. In addition, the adequacy of (2) also devolves from studies carried on optically active molecules [17], as well as from the examination of light propagation in optically active crystals [18]. In the last decade, Bohren has used these constitutive relations to compute the scattering responses of some simple objects [14,17,19,20]. Recently, we have followed up on Bohren and investigated the interaction of electromagnetic fields with planar achiral-chiral interfaces [21] as well as with non-spherical chiral objects embedded in achiral host media [22]. We have elsewhere [23] given the *infinite medium* Green's dyadic [24] and derived [23] a mathematical statement of the Huyghens' principle, and utilized [23] them to formulate scattering and radiation formalisms pertinent to these media. The present communication reports the continuation of our work in this area and we shall be considering the sources of EM fields in isotropic chiral media as well as some characteristics of the radiated fields.

In an unbounded, source-free region occupied by an isotropic chiral medium, it can be shown that all four fields -- \mathbf{E} , \mathbf{H} , \mathbf{D} , \mathbf{B} -- are divergenceless, and

$$\nabla \times \mathbf{E} = \gamma^2 \beta \mathbf{E} + i\omega\mu (\gamma/k)^2 \mathbf{H}; \quad \nabla \times \mathbf{H} = \gamma^2 \beta \mathbf{H} - i\omega\epsilon (\gamma/k)^2 \mathbf{E}, \quad (3a,b)$$

$$\nabla \times \mathbf{D} = \gamma^2 \beta \mathbf{D} + i\omega\epsilon (\gamma/k)^2 \mathbf{B}; \quad \nabla \times \mathbf{B} = \gamma^2 \beta \mathbf{B} - i\omega\mu (\gamma/k)^2 \mathbf{D}, \quad (3c,d)$$

in which the parameters k and γ are given by

$$k = \omega[\epsilon\mu]^{1/2}; \quad \gamma^2 = k^2 [1 - k^2\beta^2]^{-1}. \quad (4a,b)$$

To be noted here is the fact that k does not represent any wavenumber unless $\beta = 0$; usually, for chiral media, $|k\beta| < 1$. Furthermore, all the four field vectors follow the same governing differential equation [23],

$$\nabla^2 \mathbf{U} + 2\gamma^2 \beta \nabla \times \mathbf{U} + \gamma^2 \mathbf{U} = 0; \quad \mathbf{U} = \mathbf{E}, \mathbf{H}, \mathbf{D}, \mathbf{B}. \quad (5)$$

which reduces to the vector Helmholtz equation, $\nabla^2 \mathbf{U} + k^2 \mathbf{U} = 0$, when $\beta = 0$. Corresponding to (5), the *infinite medium* dyadic Green's function, $\mathbb{G}(\mathbf{x}, \mathbf{x}')$, is given as [23]

$$\mathbb{G}(\mathbf{x}, \mathbf{x}') = \mathbb{G}_1(\mathbf{x}, \mathbf{x}') + \mathbb{G}_2(\mathbf{x}, \mathbf{x}'), \quad (6a)$$

$$\mathbb{G}_1(\mathbf{x}, \mathbf{x}') = (k/8\pi\gamma^2) [\gamma_1 \mathbb{I} + \gamma_1^{-1} \nabla \nabla + \nabla \times \mathbb{I}] g(\gamma_1; R), \quad (6b)$$

$$\mathbb{G}_2(\mathbf{x}, \mathbf{x}') = (k/8\pi\gamma^2) [\gamma_2 \mathbb{I} + \gamma_2^{-1} \nabla \nabla - \nabla \times \mathbb{I}] g(\gamma_2; R), \quad (6c)$$

with \mathbb{I} being the unit dyadic, $\gamma_1 = k[1-k\beta]^{-1}$, $\gamma_2 = k[1+k\beta]^{-1}$, $g(\kappa; R) = \exp[i\kappa R]/R$, and $\mathbf{R} = \mathbf{x} - \mathbf{x}'$.

As a consequence of (6), the isotropic chiral medium must exhibit birefringence, *vide* the wavenumbers γ_1 and γ_2 . Linearly polarized waves cannot exist in such a medium, and the EM wave consists, in general, of a left-circularly polarized (LCP) part and a right-circularly polarized (RCP) part. If β is assumed to be positive (right-handed medium), then the LCP component propagates with the slower phase velocity, ω/γ_1 ; or else, the RCP component, with a phase velocity ω/γ_2 , is the slower one.

2. SOURCES OF LCP AND RCP FIELDS

On utilizing Maxwell's equations as well as the Green's dyadic, it can be shown that the fields radiated by electric and magnetic current densities, \mathbf{J} and \mathbf{K} , are given by

$$(k/\gamma)^2 \mathbf{E}(\mathbf{x}) = i\omega\mu \iiint d^3x' [\mathbb{I} + \beta \nabla \times \mathbb{I}] \cdot \mathbb{G}(\mathbf{x}, \mathbf{x}') \cdot \mathbf{J}(\mathbf{x}') - \iiint d^3x' [\nabla \times \mathbb{I}] \cdot \mathbb{G}(\mathbf{x}, \mathbf{x}') \cdot \mathbf{K}(\mathbf{x}'), \quad (7a)$$

$$(k/\gamma)^2 \mathbf{H}(\mathbf{x}) = i\omega\epsilon \iiint d^3x' [\mathbb{I} + \beta \nabla \times \mathbb{I}] \cdot \mathbb{G}(\mathbf{x}, \mathbf{x}') \cdot \mathbf{K}(\mathbf{x}') + \iiint d^3x' [\nabla \times \mathbb{I}] \cdot \mathbb{G}(\mathbf{x}, \mathbf{x}') \cdot \mathbf{J}(\mathbf{x}'), \quad (7b)$$

in which the integrations hold over the current-carrying volumes. Next, after noting that

$$\nabla \times \mathbb{G}_1(\mathbf{x}, \mathbf{x}') = \gamma_1 \mathbb{G}_1(\mathbf{x}, \mathbf{x}'); \quad \nabla \times \mathbb{G}_2(\mathbf{x}, \mathbf{x}') = -\gamma_2 \mathbb{G}_2(\mathbf{x}, \mathbf{x}'), \quad (8a,b)$$

it is possible to restate (7) in a form which clearly brings out the chiral flavor of the medium; *viz.*,

$$(k/\gamma)^2 \mathbf{E}(\mathbf{x}) = \gamma_1 \iiint d^3x' \mathbb{G}_1(\mathbf{x}, \mathbf{x}') \cdot [(i\omega\mu/k) \mathbf{J}(\mathbf{x}') - \mathbf{K}(\mathbf{x}')] + \gamma_2 \iiint d^3x' \mathbb{G}_2(\mathbf{x}, \mathbf{x}') \cdot [(i\omega\mu/k) \mathbf{J}(\mathbf{x}') + \mathbf{K}(\mathbf{x}')], \quad (8a)$$

$$\begin{aligned}
 (i\omega\mu/k) (k/\gamma)^2 \mathbf{H}(\mathbf{x}) = & \gamma_1 \iiint d^3x' \mathbb{B}_1(\mathbf{x}, \mathbf{x}') \cdot [(i\omega\mu/k)\mathbf{J}(\mathbf{x}') - \mathbf{K}(\mathbf{x}')] - \\
 & - \gamma_2 \iiint d^3x' \mathbb{B}_2(\mathbf{x}, \mathbf{x}') \cdot [(i\omega\mu/k)\mathbf{J}(\mathbf{x}') + \mathbf{K}(\mathbf{x}')]. \quad (8b)
 \end{aligned}$$

Thus, if in a source volume, $(i\omega\mu/k)\mathbf{J}(\mathbf{x}) + \mathbf{K}(\mathbf{x}) = 0$ then the radiated fields are purely LCP and $\mathbf{E}(\mathbf{x}) = (i\omega\mu/k)\mathbf{H}(\mathbf{x})$. On the other hand, should $(i\omega\mu/k)\mathbf{J}(\mathbf{x}) - \mathbf{K}(\mathbf{x}) = 0$ then the radiated fields are purely RCP and $\mathbf{E}(\mathbf{x}) = -(i\omega\mu/k)\mathbf{H}(\mathbf{x})$. Incidentally, these are also precisely the conditions for the generation of the circularly polarized waves in isotropic *achiral* media, as pointed out to by Rumsey [25].

The equations (8) can be even more lucidly stated if we use two auxiliary field vectors -- a LCP field vector \mathbf{Q}_1 and a RCP field vector \mathbf{Q}_2 , where [14]

$$\mathbf{Q}_1 = (1/2) [\mathbf{E} + (i\omega\mu/k)\mathbf{H}]; \quad \mathbf{Q}_2 = (1/2)[(i\omega\epsilon/k)\mathbf{E} + \mathbf{H}]. \quad (9a,b)$$

In that case,

$$(k/\gamma)^2 \mathbf{Q}_1(\mathbf{x}) = \gamma_1 \iiint d^3x' \mathbb{B}_1(\mathbf{x}, \mathbf{x}') \cdot [(i\omega\mu/k)\mathbf{J}(\mathbf{x}') - \mathbf{K}(\mathbf{x}')], \quad (10a)$$

$$(k/\gamma)^2 \mathbf{Q}_2(\mathbf{x}) = - (i\omega\mu/k) \gamma_2 \iiint d^3x' \mathbb{B}_2(\mathbf{x}, \mathbf{x}') \cdot [(i\omega\mu/k)\mathbf{J}(\mathbf{x}') + \mathbf{K}(\mathbf{x}')]. \quad (10b)$$

As can be observed from (10), the radiation of \mathbf{Q}_1 is independent from that of \mathbf{Q}_2 . The coupling between the LCP and the RCP fields takes place only at bimaterial boundaries [22] where conditions on the tangential components of \mathbf{E} and \mathbf{H} must be satisfied, i.e., the boundary conditions are specified not on the \mathbf{Q} 's singly, but on the combinations $[\mathbf{Q}_1 - (i\omega\mu/k)\mathbf{Q}_2]$ and $[\mathbf{Q}_2 - (i\omega\epsilon/k)\mathbf{Q}_1]$.

3. RADIAL AND TRANSVERSE FIELDS

Reverting to the use of \mathbf{E} and \mathbf{H} , it can be shown using the constitutive equations (2) as well as Maxwell's equations that

$$(k/\gamma)^2 \nabla \times \mathbf{E} = i\omega\mu\mathbf{H} + k^2\beta\mathbf{E} + (i\omega\mu\beta\mathbf{J} - \mathbf{K}), \quad (11a)$$

$$(k/\gamma)^2 \nabla \times \mathbf{H} = -i\omega\epsilon\mathbf{E} + k^2\beta\mathbf{H} + (i\omega\epsilon\beta\mathbf{K} + \mathbf{J}). \quad (11b)$$

The cross-product of both of these equations is taken with $\mathbf{r} = r\mathbf{e}_r$ and the transverse components (subscripted t) of the resulting equations collected. Then, since

$$(\mathbf{r} \times \nabla \times \mathbf{A})_t = r^{-1} \text{grad}_{\theta\phi}(\mathbf{r} \cdot \mathbf{A}) - r\{\partial/\partial r\}\mathbf{A}_t - \mathbf{A}_t, \quad (12a)$$

$$\text{grad}_{\theta\phi} = \mathbf{e}_\theta \{\partial/\partial\theta\} + \mathbf{e}_\phi (1/\sin\theta)\{\partial/\partial\phi\}, \quad (12b)$$

in a spherical co-ordinate system, it follows that

$$(k/\gamma)^2 r^{-1} \text{grad}_{\theta\phi}(\mathbf{r} \cdot \mathbf{E}) - (k/\gamma)^2 \{\partial/\partial r\}(r\mathbf{E}_t) - i\omega\mu \mathbf{r} \times (\mathbf{H} - i\omega\epsilon\beta\mathbf{E}) = \mathbf{r} \times (i\omega\mu\beta\mathbf{J} - \mathbf{K}), \quad (13a)$$

$$(k/\gamma)^2 r^{-1} \text{grad}_{\theta\phi}(\mathbf{r} \cdot \mathbf{H}) - (k/\gamma)^2 \{\partial/\partial r\}(r\mathbf{H}_t) + i\omega\epsilon \mathbf{r} \times (\mathbf{E} + i\omega\mu\beta\mathbf{H}) = \mathbf{r} \times (i\omega\epsilon\beta\mathbf{K} + \mathbf{J}). \quad (13b)$$

In order to eliminate the radial components in (13a,b), we go back to (11a,b) and take their dot-products with \mathbf{r} . Then, it turns out that

$$i\omega\epsilon\mathbf{r} \cdot \mathbf{E} = \mathbf{r} \cdot \mathbf{J} + \nabla \cdot [\mathbf{r} \times (\mathbf{H} + i\omega\epsilon\beta\mathbf{E})], \quad (14a)$$

$$i\omega\mu\mathbf{r} \cdot \mathbf{H} = \mathbf{r} \cdot \mathbf{K} - \nabla \cdot [\mathbf{r} \times (\mathbf{E} - i\omega\mu\beta\mathbf{H})]. \quad (14b)$$

Substitution of (14) in (13) finally leads to two differential equations for the transverse components of the fields:

$$-(k/\gamma)^2 (i\omega\epsilon r^2)^{-1} \text{grad}_{\theta\phi} \text{div}_{\theta\phi} [\mathbf{r} \times (\mathbf{H} + i\omega\epsilon\beta\mathbf{E})] - (k/\gamma)^2 \{\partial/\partial r\} (r\mathbf{E}_t) - i\omega\mu \mathbf{r} \times (\mathbf{H} - i\omega\epsilon\beta\mathbf{E}) \\ = \mathbf{r} \times (i\omega\mu\beta\mathbf{J} - \mathbf{K}) - (k/\gamma)^2 (i\omega\epsilon)^{-1} \text{grad}_{\theta\phi} [\mathbf{e}_r \cdot \mathbf{J}], \quad (15a)$$

$$-(k/\gamma)^2 (i\omega\mu r^2)^{-1} \text{grad}_{\theta\phi} \text{div}_{\theta\phi} [\mathbf{r} \times (\mathbf{E} - i\omega\mu\beta\mathbf{H})] - (k/\gamma)^2 \{\partial/\partial r\} (r\mathbf{H}_t) + i\omega\epsilon \mathbf{r} \times (\mathbf{E} + i\omega\mu\beta\mathbf{H}) \\ = \mathbf{r} \times (i\omega\epsilon\beta\mathbf{K} + \mathbf{J}) - (k/\gamma)^2 (i\omega\mu)^{-1} \text{grad}_{\theta\phi} [\mathbf{e}_r \cdot \mathbf{K}]. \quad (15b)$$

Separation of \mathbf{E} and \mathbf{H} from (15a,b) is well nigh impossible, and once again, the auxiliary fields \mathbf{Q}_1 and \mathbf{Q}_2 serve us in good stead. It turns out that

$$r^{-2} \text{grad}_{\theta\phi} \text{div}_{\theta\phi} [\mathbf{r} \times \mathbf{Q}_1] + \{\partial/\partial r\} (r\mathbf{Q}_{1t}) + \gamma_1 \mathbf{r} \times \mathbf{Q}_1 \\ = -(1/2) (\gamma_1/k) [\mathbf{r} \times (i\omega\mu\mathbf{J}/k - \mathbf{K}) + \text{grad}_{\theta\phi} [\mathbf{e}_r \cdot (i\omega\mu\mathbf{J}/k - \mathbf{K})]], \quad (16a)$$

$$r^{-2} \text{grad}_{\theta\phi} \text{div}_{\theta\phi} [\mathbf{r} \times \mathbf{Q}_2] - \{\partial/\partial r\} (r\mathbf{Q}_{2t}) + \gamma_2 \mathbf{r} \times \mathbf{Q}_2 \\ = (1/2) (\gamma_2/k) (k/i\omega\mu) [\mathbf{r} \times (i\omega\mu\mathbf{J}/k + \mathbf{K}) + \text{grad}_{\theta\phi} [\mathbf{e}_r \cdot (i\omega\mu\mathbf{J}/k + \mathbf{K})]], \quad (16b)$$

in which equations the divergence operator is defined by

$$\text{div}_{\theta\phi} \mathbf{A} = (1/\sin\theta) [\{\partial/\partial\theta\}(\sin\theta A_\theta) + \{\partial A_\phi/\partial\phi\}]. \quad (17)$$

4. EIGENFUNCTION EXPANSIONS OF \mathbf{Q}_1 AND \mathbf{Q}_2

Equations (10) and (16) are of great import because they allow independent eigenfunction expansions of \mathbf{Q}_1 and \mathbf{Q}_2 . In order to do so, we note the Helmholtz theorem on the sphere: Provided \mathbf{A} is a field of vectors on a sphere and is of class C^3 , then there exist functions $F(\mathbf{x})$, $S(\mathbf{x})$ and $T(\mathbf{x})$ such that

$$\mathbf{A} = F\mathbf{e}_r + \text{grad}_{\theta\phi} S + \mathbf{e}_r \times \text{grad}_{\theta\phi} T, \quad (18)$$

pursuant to certain conditions on these scalars, which have been discussed at length by van Bladel [26]. As a result [27], it is possible to express the *divergenceless* fields \mathbf{Q}_1 and \mathbf{Q}_2 in a source-free region as

$$\mathbf{Q}_1 = -(i\omega\epsilon)^{-1} \nabla \times \nabla \times (\mathbf{u}_1 \mathbf{r}) + \nabla \times (\mathbf{v}_1 \mathbf{r}); \quad (19a,b)$$

$$\mathbf{Q}_2 = -(i\omega\epsilon)^{-1} \nabla \times \nabla \times (\mathbf{u}_2 \mathbf{r}) + \nabla \times (\mathbf{v}_2 \mathbf{r}),$$

in which the scalars $u_1(\mathbf{x}) = u_1(r, \theta, \phi)$, etc. must satisfy the scalar Helmholtz equations

$$[\nabla^2 + \gamma_1^2] u_1 = 0; \quad [\nabla^2 + \gamma_1^2] v_1 = 0; \quad (20a,b)$$

$$[\nabla^2 + \gamma_2^2] u_2 = 0; \quad [\nabla^2 + \gamma_2^2] v_2 = 0; \quad (20c,d)$$

along with the integrability conditions

$$\int_0^\pi d\theta \sin\theta \int_0^{2\pi} d\phi u_1 = 0, \text{ etc.} \quad (21)$$

It can be easily verified that the scalar spherical harmonics, $z_n(\gamma_1 r) P_n^m(\cos\theta) \exp[i m \phi]$ and $z_n(\gamma_2 r) P_n^m(\cos\theta) \exp[i m \phi]$, excluding $n=m=0$, satisfy the conditions (20) and (21). Consequently, if we choose

$$v_1 = z_n(\gamma_1 r) P_n^m(\cos\theta) \exp[i m \phi]; \quad u_1 = -(i\omega\epsilon/\gamma_1) v_1, \quad (22a,b)$$

$$v_2 = z_n(\gamma_2 r) P_n^m(\cos\theta) \exp[i m \phi]; \quad u_2 = (i\omega\epsilon/\gamma_2) v_2, \quad (22c,d)$$

then it is possible to expand Q_1 and Q_2 in terms of the vector spherical harmonics defined by Stratton[28] as

$$Q_1(r) = \sum_{\sigma mn} a_{1\sigma mn} [M_{\sigma mn}(\gamma_1 r) + N_{\sigma mn}(\gamma_1 r)]; \quad (23a)$$

$$Q_2(r) = \sum_{\sigma mn} a_{2\sigma mn} [M_{\sigma mn}(\gamma_2 r) - N_{\sigma mn}(\gamma_2 r)]. \quad (23b)$$

In this discussion, z_n are the appropriate spherical Bessel functions, P_n^m are the associated Legendre polynomials and $a_{1\sigma mn}$ and $a_{2\sigma mn}$ are the expansion coefficients. The specific forms of u_1 , etc. chosen in (22) ensure that while

$$[\nabla^2 + \gamma_1^2]Q_1 = 0; \quad [\nabla^2 + \gamma_2^2]Q_2 = 0, \quad (24a,b)$$

the circularly polarized characters of Q_1 and Q_2 are maintained *vide*

$$\nabla \cdot Q_1 = \gamma_1 Q_1; \quad \nabla \cdot Q_2 = -\gamma_2 Q_1. \quad (25a,b)$$

5. RADIATION FROM SOME SIMPLE SOURCES

Finally, we consider here the radiation fields of some simple sources and show how their radiation characteristics are different in chiral media from those in achiral ones. We begin by considering an electric dipole source \mathbf{p} located at the origin. In this case, $\mathbf{J}(\mathbf{x}) = -i\omega\mathbf{p}\delta(\mathbf{x})$, and from (7a,b), it is easy to see that the radiated fields are given by

$$\mathbf{E}(\mathbf{x}) = (\omega^2\mu/k) (\gamma/k)^2 [\gamma_1 \mathbb{G}_1(\mathbf{x}, 0) + \gamma_2 \mathbb{G}_2(\mathbf{x}, 0)] \cdot \mathbf{p}, \quad (26a)$$

$$\mathbf{H}(\mathbf{x}) = (-i\omega) (\gamma/k)^2 [\gamma_1 \mathbb{G}_1(\mathbf{x}, 0) - \gamma_2 \mathbb{G}_2(\mathbf{x}, 0)] \cdot \mathbf{p}, \quad (26b)$$

$\mathbf{0}$ being the null vector. Likewise, for a magnetic dipole \mathbf{m} located at the origin, $\mathbf{J}(\mathbf{x}) = \nabla \times [\mathbf{m}\delta(\mathbf{x})]$, and the radiated fields turn out to be

$$\mathbf{E}(\mathbf{x}) = (i\omega\mu/k) (\gamma/k)^2 [\gamma_1^2 \mathbb{G}_1(\mathbf{x}, 0) - \gamma_2^2 \mathbb{G}_2(\mathbf{x}, 0)] \cdot \mathbf{m}, \quad (27a)$$

$$\mathbf{H}(\mathbf{x}) = (\gamma/k)^2 [\gamma_1^2 \mathbb{G}_1(\mathbf{x}, 0) + \gamma_2^2 \mathbb{G}_2(\mathbf{x}, 0)] \cdot \mathbf{m}. \quad (27b)$$

The far zone radiated fields for both \mathbf{p} and \mathbf{m} can be computed easily by noting that when $\gamma_s r \gg 1$, ($s = 1, 2$), the Green's dyadics can be approximated by

$$\mathbb{G}_s(\mathbf{x}, \mathbf{x}') = (ik\gamma_s/8\pi\gamma^2) \mathbb{D}_s(\mathbf{x}) g(\gamma_s; r) \exp[-i\gamma_s \mathbf{e}_r \cdot \mathbf{x}'], \quad (28)$$

where the dyadics

$$\mathbb{D}_1(\mathbf{x}) = \mathbf{e}_r \times [\mathbf{i}\mathbf{e}_r \times \mathbf{I} + \mathbf{I}]; \quad \mathbb{D}_2(\mathbf{x}) = \mathbf{e}_r \times [\mathbf{i}\mathbf{e}_r \times \mathbf{I} - \mathbf{I}]. \quad (29a,b)$$

The important difference between chiral and achiral media can be easily seen now by examining (26) and (27). Without loss of generality, let the source dipole moments be parallel to the z axis. Then, if the dipole moments were to be radiating in an achiral medium, at $\mathbf{x} = z\mathbf{e}_z$ there is no H-field due to \mathbf{p} and there is no E-field due to \mathbf{m} . On the other hand, the phase differences between the LCP and RCP components guarantee that in a chiral medium, both E- and H- fields exist on the z axis regardless of which dipole moment is radiating.

A similar conclusion can be drawn if the source is a constant-current loop of radius a . Therefore, $\mathbf{J}(\mathbf{x}) = (I_0/a)\mathbf{e}_\phi \delta(r-a)\delta(\theta-\pi/2)$. Using (7a,b), the radiation field on the z axis can be worked out to yield

$$\mathbf{E}(ze_z) = \mathbf{e}_z (kI_0\pi a^2)(i\omega\mu/k) [(\gamma_1/k)^2 h(\gamma_1; R_a) - (\gamma_2/k)^2 h(\gamma_2; R_a)], \quad (30a)$$

$$\mathbf{H}(ze_z) = \mathbf{e}_z (kI_0\pi a^2) [(\gamma_1/k)^2 h(\gamma_1; R_a) + (\gamma_2/k)^2 h(\gamma_2; R_a)], \quad (30b)$$

in which $h(\kappa; R) = [(iR)^{-1} + (\kappa R^2)^{-1}]g(\kappa; R)$ and $R_a = \sqrt{a^2 + z^2}$. It is easy to see that if $\beta = 0$, then the right side of (30a) reduces identically to zero, as would be expected.

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