Engineering Mechanics 12: *Dynamics*
Sections 6 & 7
Course Information
Fall 2000

Professors

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Time/Place

Section 6 (Prof. Costanzo): MWF 1:25–2:15, 120 Earth-Engineering Sciences Bldg.
Section 7 (Prof. Gray): MWF 2:30–3:20, 120 Earth-Engineering Sciences Bldg.

Office Hours

Professor Gray:
MW 3:30–4:30 a.m. or by appointment. To make an appointment, please see me at class time or contact me via email.

Professor Costanzo:
MW 3:30–4:30 a.m. or by appointment. To make an appointment, please see me at class time or contact me via email.

Textbook

Pytel, Andrew and Jaan Kiusalaas (1999) *Engineering Mechanics: Dynamics*, 2nd Edition, Brooks/Cole Publishing, Pacific Grove, California.

Objectives

The primary objective in any introductory mechanics course is to develop in the engineering student the ability to analyze any problem in a simple and logical manner and to apply to its solution a few, well-understood, basic principles. This objective will be obtained by developing good problem solving skills (think before beginning the solution, ask what principles apply and critically judge your results), visualization skills (good free-body diagrams), and emphasizing basic principles (dynamics is not a sequence of independent methods for solving problems, but is a coherent class of techniques all based on Newton's laws).

Expectations

In line with Penn State University's educational mission, both professors and students should be committed to nothing less than *academic excellence*. Therefore, just as the professor is responsible for attending every class period, so are you, the student, also responsible for doing so. Therefore, you are responsible for everything covered or announced in *every* class period. There are no exceptions.

In addition, each student is expected to have a working knowledge of the material covered in *all* prerequisite courses. That means you should know:

- Geometry and trigonometry, including the laws of sines and cosines, direction cosines, and the like.

- Statics. That is, how to draw correct free-body diagrams, find force and moment equilibrium, write friction laws, find support reactions, and find moments of inertia.
- Differential and integral calculus. That is, how to differentiate and integrate most simple functions, simple sequences and series.

You are also expected to devote sufficient time to master the course material. It is unreasonable to expect that good performance can be achieved without study and that is especially true for this course. The general "rule of thumb" stating that the student should devote **at least** two hours outside of class for every hour in class certainly applies to EMch 12, though you will probably find this course to be more difficult than most.

Grading Your grade will be decided based on the following scheme:

Category	*Percent of Grade*
Midterm Exams (3)	19% per exam
Homework	15%
Final Exam	28%

Your grade will be determined by your performance. The latter will be evaluated without reference to that of your classmates. In other words, your performance will be evaluated using *objective* standards rather than standards based on a notion of average class performance (*i.e.*, **we do not grade on a curve**). Your grade will be based on a scale of 100 points. The following chart provides letter grade equivalents of your numerical grade:

A	A–	B+	B	B–	C+	C	D	F
93–100	90–93	87–90	82–87	79–82	76–79	68–76	60–68	< 60

Given this information, it should be apparent that you will never need to ask about your grade at any time during the semester, since you will always have all the information to compute it yourself.

Exams The midterms will be given in the evening on the dates shown on the syllabus. In the event you are unable to attend one of the scheduled exams, **NO MAKEUP EXAM WILL BE ADMINISTERED** unless **ALL** of the following conditions are met:

1. **Legitimate Reason**
   The missed exam is caused by circumstances beyond the student's control, such as illness, family emergency, or a university-sponsored activity.
2. **Prior Notification**
   It is your responsibility to notify us **prior** to the exam if you are unable to attend (this may be done in person, by phone, or by email). If circumstances prevent you from contacting us directly, then you may also notify the Assistance Center at (814) 863-2020, which will, in turn, notify our department, Engineering Science and Mechanics.

3. **Written Verification**
   Sufficient information must be provided so that your claim can be verified. For example, if you miss an exam due to illness, we must receive a letter from your physician stating that you were in no condition to take an exam. A slip of paper from Ritenour stating that "you were seen" will **not** suffice.

If you miss an exam and are unable to satisfactorily fulfill each of the above three conditions, than you will receive a zero for that exam. For those cases where a makeup exam is warranted, you may either take an *oral makeup exam* or take the average of your other exams as your score (if you miss two of the three exams, then taking the average is not an option).

An equation sheet will be handed out the first day of class. You will be allowed to use it for each exam and for the final exam. Please let us know if there is anything you would like to see added to the equation sheet—we will grant all reasonable requests.

Homework You will receive your homework assignments by email every Friday. At the *beginning* of every Friday class, one of you will roll a die to randomly determine which problem you will turn in that week. It is vital that you ask questions during class or office hours if you do not understand how to do a homework problem.

General Info

- Bring your textbook to *every* class. We will work many example problems out of the text and you will not be able to keep up if you don't know what problem we are working.
- We cannot emphasize enough how important it is for you to attempt all the homework and keep pace with the class. If you do not keep pace with the class, attempt the homework, and regularly attend class you will have a very difficult time. We would like to see every student do the following:
  1. Read the section that is to be covered in class *before* the class. Also take a look at the assigned homework problems before attending class.
  2. Attend class and see what we have to say about the current topic. Doing Step 1 will help you understand the examples we work in class.
  3. Read the section one more time after class and then try the homework problems. See us if there are problems you can't get after an honest effort.
- Finally, you will benefit in this class, and in all your other classes, by learning a software package such as Mathematica, Maple, or Mathcad. Each of these packages allow you to easily solve systems of linear and nonlinear algebraic equations and/or systems of linear and nonlinear differential equations such as those that arise in dynamics. In addition, each of these packages can do so both numerically and symbolically. By learning one of these packages (and using it) you will be able to focus on understanding dynamics (*i.e.*, setting up problems and writing governing equations) rather than doing tedious algebra to make sure you have set the problem up correctly.

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Course Syllabus: Sections 6 & 7, Fall 2000

Exam	*Date*	*Location*
Exam 1	Tuesday, September 19, 6:30–7:45 p.m.	115 EE West
Exam 2	Tuesday, October 17, 6:30–7:45 p.m.	115 EE West
Exam 3	Tuesday, November 21, 6:30–7:45 p.m.	115 EE West
Final Exam	Time and date will be announced.	TBD

Reading	**Topic**
11	Course Introduction and Mathematical Prelimaries (vectors, calculus, and more)
	Particle Dynamics
12.1–12.5	Kinematics, Rectangular Coordinates, and Rectilinear Equations of Motion/Kinetics
13.1–13.2	Kinematics in Normal-Tangential or Path Coordinates
13.3	Kinematics in Polar or Cylindrical Coordinates
13.4	Equations of Motion in Curvilinear Coordinates (Normal-Tangential and Polar)
14.1–14.4	Work-Energy Principle for a Particle, Conservative Forces, and Conservation of Energy
14.6	Impulse-Momentum Principle for a Particle
14.7	Angular Impulse-Momentum Principle for a Particle
15.1–15.3	Kinematics of Relative and Constrained Motion
15.4	Equations of Motion for Systems of Particles; The Mass Center
15.5	Work-Energy Principles for Systems of Particles
15.6–15.7	Linear and Angular Impulse-Momentum Principles for Systems of Particles
15.8–15.9	Plastic Impact and Impulsive Motion
15.10	Elastic Impact
	Rigid Body Dynamics
16.1–16.3	Kinematics of Pure Translation and Rotation About a Fixed Axis
16.4–16.5	Rigid Body Velocity Analysis
16.6	Instantaneous Center of Zero Velocity
16.7	Rigid Body Acceleration Analysis
17.1–17.2	Mass Moments of Inertia
17.3–17.5	Angular Momentum of a Rigid Body and Equations of Motion
18.1–18.4	Work of a Couple; Kinetic Energy of a Rigid Body; W-E Principle for Rigid Bodies
18.5–18.6	Linear and Angular Impulse-Momentum Methods for Rigid Bodies
	Introduction to Vibrations/Oscillations
20.1–20.2	Undamped Free Vibrations of Particles
20.3	Undamped Forced Vibrations of Particles
20.4	Damped Free Vibrations of Particles

Equation Sheet for Engineering Mechanics 12—Dynamics

Note: vectors are indicated by **boldface type**.

Miscellaneous

If $ax^2 + bx + c = 0$, then $x = \left(-b \pm \sqrt{b^2 - 4ac}\right) \Big/ 2a$.

Rectilinear (1-D) Motion

Position: $s(t)$; Velocity: $v = \dot{s} = ds/dt$; Acceleration: $a = \ddot{s} = dv/dt = d^2s/dt^2 = v dv/dx$. For constant acceleration a_c:

$$v^2 = v_0^2 + 2a_c(s - s_0) \qquad v = v_0 + a_c t \qquad s = s_0 + v_0 t + \tfrac{1}{2} a_c t^2$$

2D Motions—Cartesian Coordinates

Position: $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$; Velocity: $\mathbf{v} = d\mathbf{r}/dt = \dot{x}\hat{\mathbf{i}} + \dot{y}\hat{\mathbf{j}}$; Acceleration: $\mathbf{a} = d\mathbf{v}/dt = d^2\mathbf{r}/dt^2 = \ddot{x}\hat{\mathbf{i}} + \ddot{y}\hat{\mathbf{j}}$

2D Motions—Normal-Tangential (Path) Coordinates

$$\mathbf{v} = v\hat{\mathbf{e}}_t \qquad v = \rho\dot{\theta} \qquad \mathbf{a} = a_t\hat{\mathbf{e}}_t + a_n\hat{\mathbf{e}}_n \qquad a_t = \dot{v} \qquad a_n = \rho\dot{\theta}^2 = v^2/\rho$$

2D Motions—Polar Coordinates

$$\mathbf{r} = r\hat{\mathbf{e}}_r \qquad \mathbf{v} = v_r\hat{\mathbf{e}}_r + v_\theta\hat{\mathbf{e}}_\theta \qquad v_r = \dot{r} \qquad v_\theta = r\dot{\theta}$$

$$\mathbf{a} = a_r\hat{\mathbf{e}}_r + a_\theta\hat{\mathbf{e}}_\theta \qquad a_r = \ddot{r} - r\dot{\theta}^2 \qquad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

Newton's Second Law

$\sum \mathbf{F} = m\mathbf{a}$; $\sum F_x = ma_x$; $\sum F_y = ma_y$; $\sum F_n = ma_n$; $\sum F_t = ma_t$; $\sum F_r = ma_r$; $\sum F_\theta = ma_\theta$

Work-Energy Principle

$$U_{1\text{–}2} = \int_{\mathcal{L}} \mathbf{F} \cdot d\mathbf{r} \qquad T = \tfrac{1}{2}mv^2 \qquad T_1 + \sum U_{1\text{–}2} = T_2$$

$$V_g = Wy \qquad T_1 + V_1 + (U_{1\text{–}2})_{\text{nc}} = T_2 + V_2 \qquad V_e = \tfrac{1}{2}k\delta^2$$

Linear Impulse-Momentum Principle

$$\mathbf{p} = m\mathbf{v} \qquad \mathbf{F} = \frac{d\mathbf{p}}{dt} \qquad \mathbf{L}_{1\text{–}2} = \int_{t_1}^{t_2} \mathbf{F}\, dt = m\mathbf{v}_2 - m\mathbf{v}_1$$

Impact of Smooth Particles

For a coefficient of restitution e, in the direction along to the line of impact (LOI) or n direction:

$$e = \frac{v_{\text{separate}}}{v_{\text{approach}}} = \frac{v_{B_2} - v_{A_2}}{v_{A_1} - v_{B_1}}$$

Total linear momentum is conserved in the n direction: $(m_A v_{A_1} + m_B v_{B_1} = m_A v_{A_2} + m_B v_{B_2})_n$. Velocity of each particle is conserved perpendicular to the LOI (t direction):$(v_{A_1} = v_{A_2})_t$ and $(v_{B_1} = v_{B_2})_t$.

Angular Impulse-Momentum Principle

$$\mathbf{h}_A = \mathbf{r}_{P/A} \times m\mathbf{v} \qquad \mathbf{M}_A = \dot{\mathbf{h}}_A + \mathbf{v}_A \times m\mathbf{v}_G \qquad h_O = mr^2\dot{\theta}$$

$$(\mathbf{A}_A)_{1-2} = \int_{t_1}^{t_2} \mathbf{r} \times \mathbf{F}\, dt = \int_{t_1}^{t_2} \mathbf{M}_A\, dt = \mathbf{h}_2 - \mathbf{h}_1$$

Rigid Body Kinematics

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} \qquad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A} \qquad \mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha} \times \mathbf{r}_{B/A} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{B/A})$$

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\Omega} \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$$ where xyz refers to the rotating frame ($\mathcal{B}$)

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{B/A} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{B/A}) + (\mathbf{a}_{B/A})_{xyz} + 2\boldsymbol{\Omega} \times (\mathbf{v}_{B/A})_{xyz}$$

Moments of Inertia

$$(I_G)_{\text{disk}} = \tfrac{1}{2}mr^2 \qquad (I_G)_{\text{rod}} = \tfrac{1}{12}ml^2 \qquad (I_G)_{\text{plate}} = \tfrac{1}{12}m(a^2+b^2) \qquad (I_G)_{\text{sphere}} = \tfrac{2}{5}mr^2$$

Parallel Axis Theorem: $I_A = I_G + md^2$; Radius of Gyration: $I_A = mk_A^2$

Angular Momentum and Equations of Motion for a Rigid Body

Angular Momentum

$$\mathbf{h}_A = I_G\boldsymbol{\omega} + \mathbf{r}_{G/A} \times m\mathbf{v}_G \qquad \mathbf{h}_A = I_A\boldsymbol{\omega} + \mathbf{r}_{G/A} \times m\mathbf{v}_A$$

Equations of Motion

For the mass center G: $\sum M_G = I_G\alpha$ and for a fixed point O: $\sum M_O = I_O\alpha$. For an arbitrary point A:

$$\sum \mathbf{M}_A = I_G\boldsymbol{\alpha} + \mathbf{r}_{G/A} \times m\mathbf{a}_G \qquad \sum \mathbf{M}_A = I_A\boldsymbol{\alpha} + \mathbf{r}_{G/A} \times m\mathbf{a}_A \qquad (\mathbf{M}_A)_{\text{FBD}} = (\mathbf{M}_A)_{\text{MAD}}$$

Work-Energy for a Rigid Body

The work-energy principle is the same as that for particles. The kinetic energy of a rigid body is:

$$T = \tfrac{1}{2}mv_A^2 + \tfrac{1}{2}I_A\omega^2 - m\omega(\bar{y}v_{A_x} - \bar{x}v_{A_y})$$

Vibrations

Undamped free vibration: Equation of motion: $\ddot{x} + p^2x = 0$, $p = \sqrt{k/m}$, $x(t) = x_0\cos pt + (v_0/p)\sin pt$. Period: $\tau = 2\pi/p$. Frequency: $f = 1/\tau = p/2\pi$.

Damped free vibration: Equation of motion: $m\ddot{x} + c\dot{x} + kx = 0$, $\zeta = c/c_{\text{cr}} = c/(2mp)$,

$$x(t) = A_1e^{(-\zeta+\sqrt{\zeta^2-1})pt} + A_2e^{(-\zeta-\sqrt{\zeta^2-1})pt},$$

and for $\zeta < 1$: $x(t) = Ee^{-\zeta pt}\sin(\omega_d t + \alpha)$, where E and α are constants and $\omega_d = p\sqrt{1-\zeta^2}$.

Harmonic Forcing: Equation of motion: $\ddot{x} + p^2x = P_0\sin\omega t$ with solution:

$$x(t) = E\sin(pt+\alpha) + \frac{P_0/k}{1-(\omega/p)^2}\sin\omega t$$

Damped Forced Vibration: Equation of motion: $m\ddot{x} + c\dot{x} + kx = P_0\sin\omega t$ with solution: $x(t) = X\sin(\omega t + \phi)$, where

$$X = \frac{P_0/k}{\sqrt{[1-(\omega/p)^2]^2 + (2\zeta\omega/p)^2}}$$

$$\phi = \tan^{-1}\left[\frac{2\zeta\omega/p}{1-(\omega/p)^2}\right]$$